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ABSTRACT

Decreasing fiscal budgets and declining student enrollments have forced many school districts to evaluate the alternative strategies in closing certain elementary or secondary schools, either temporarily or permanently, for purposes of consolidation. For rural and other small districts, the strategy of multiple district consolidation (merger) has also been attempted in order to best allocate limited resources. A multiple-criterion model was introduced in 1978 in a major American Midwestern city for such purposes. This model has been enhanced not only to evaluate schools for the potential benefits to the district of temporary versus permanent closure, but so that it can also be used for reallocating school districts. The process is completely computer-modeled and employs standard statistical computations to assess the effect on the district of sample alternative closure schemes. A 16-item list of selected readings and references is included. (Author/TE)

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MODELING ELEMENTARY AND SECONDARY SCHOOL CLOSURES THROUGH MAM™

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International Society of Educational Planners (ISEP)
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ABSTRACT

Targeting elementary and/or secondary schools for temporary and/or permanent closure during declining enrollment or scant budgeting periods is accomplished through computer modeling.

SUMMARY

Decreasing fiscal budgets and declining student enrollments have forced many school districts to evaluate the alternative strategies in closing, either temporarily or permanently, certain elementary and/or secondary schools for the purposes of consolidation. For rural and other small districts, the strategy of multiple district consolidation (merger) has also been attempted in order to allocate most optimally the limited financial resources available. A multiple-criterion model for targeting a finite number of schools for closure was introduced in 1978 in a major American midwest city for such purposes. This model has been enhanced to not only evaluate schools for the potential benefit to the district in targeting individual school potential for temporary v. permanent closure, but can not also be utilized for reallocating student attendance districts under the rhetoric of multiple district consolidation. The process is completely computer-modeled, and employs standard statistical computations to post-assess the impact to the district of sample, alternative closure schemes.

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DEDICATION

This project is dedicated to all:

1. chief state and district school officers who are convinced that data and information should never get in the way of good decisionmaking;
2. building level principals and assistant principals who assure their classroom teachers and program coordinators that trends are never that reliable to be predictable;
3. parents and community citizens, all, who are convinced that the market value of their residential properties will decline without the immediate proximity of an elementary school in their neighborhood;
4. school board members who insist that money is not the object when declining enrollments are coincided with dwindling fiscal reserves in the district; and finally,
5. those same chief state and district school officers who, when the list of initially targeted schools for closure has been received from the evaluation team, declare that not enough data/information have been analyzed to understand the ramifications of such recommendations.

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MISSION

The purpose of this paper is to define, develop and display a mathematical modeling formulation which has significant potential for exploring the myriad of decisions available to the school district officer when faced with the need to evaluate each of the district's campus sites for potential future closure.

An appropriate mission statement of such a model for targeted school site closures would be formulated as:

To design a system of alternative school closure options, accounting for differential levels of operational discontinuance or continuance for each school site evaluated, based upon likely scenarios for the future needs and demands of the school district and its community, which could be compared and evaluated against established policy standards, and instructional and administrative requirements, and which would result in the selection of which schools would be closed or allowed to remain open, and the level of that operational discontinuance or continuance, respectively.

Thus the hierarchical school closure model will successfully determine (1) what number of schools should be targeted for closure, (2) the disposition of those closed schools, and (3) what will be the operating level of those schools remaining open.

This modeling technique allows the decisionmaker to utilize all available data and information for simulating the impact to the school district which alternative closure decisions per each individual site might provide. This forecasting strategy includes the ability to control for interrelationships between discrete pairs of programs in contrasted schools, as well as among the collective impact of groups of programs with their individual levels of recommended resource support. Finally, this modeling strategy allows the decisionmaker to vary the initial controls placed upon the model's mathematical structure, thereby simulating alternative decisional solutions based upon an ever fluid and transactional environment.

BACKGROUND

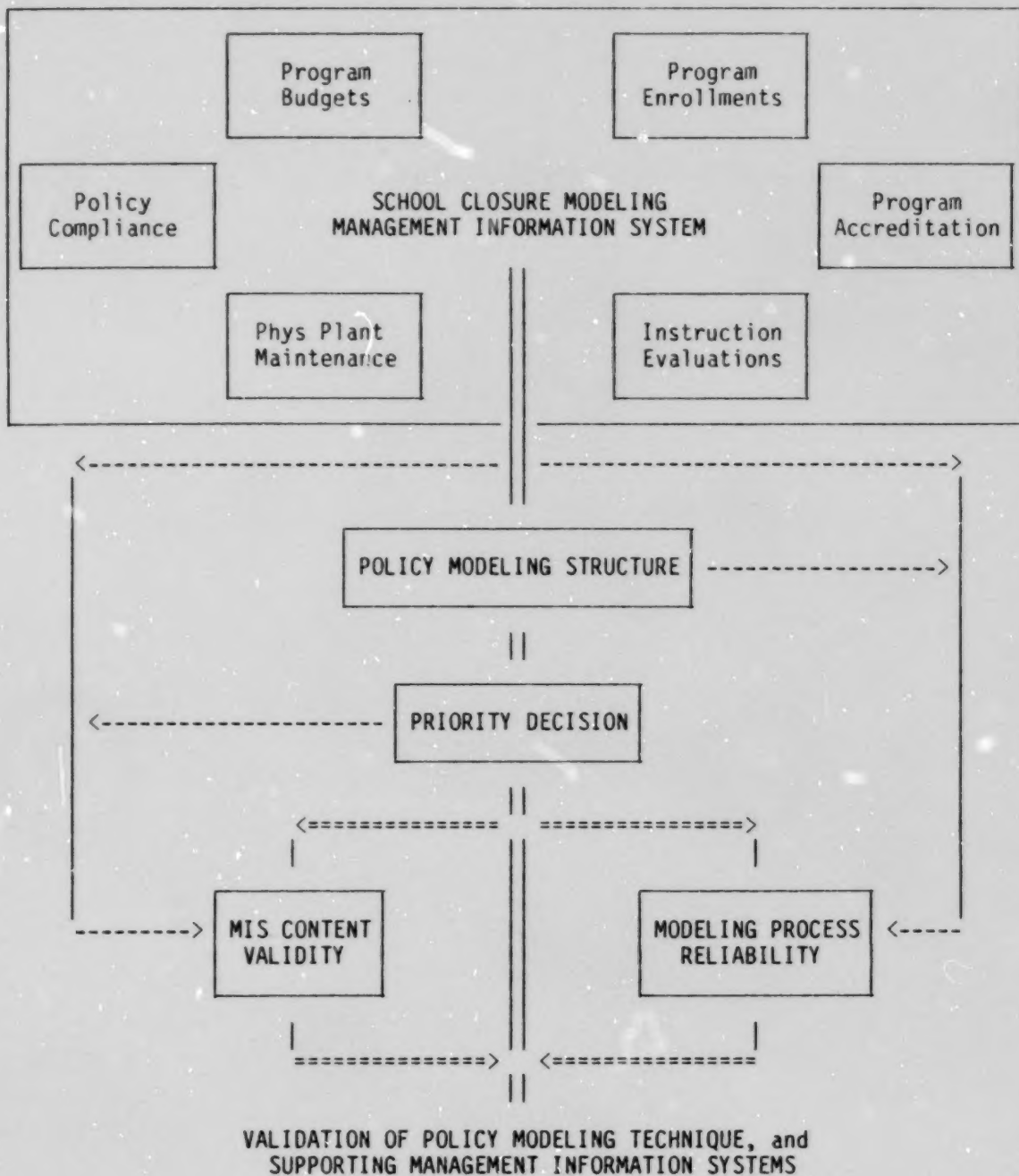
School district officers and local school boards have been faced with the issue of school closures over since the beginning decline in student enrollments during the early 1970's. With the baby boom of the late 1940's successfully completing high school in 1968-1972, school administrators looked ahead to these students' children repopulating the schools during the late 1970's and throughout the 1980's. Zero-population growth enthusiasts, the economy, and a general alteration in life style brought such hopes to a disastrous end. By the mid-1970's, many school districts had experienced nearly a 50 percent decline in their ADM (average daily membership). With the economy creating parallel problems in terms of public and private educational funding, the decision by 1977 in many school districts was to close schools. Unfortunately, no one in the position to implement such a strategy knew where to start, or how to proceed in the volatile issue of school closures.

The historical record (see The Design, Implementation and Evaluation of Mathematical Modeling Procedures for Decisioning Among Educational Alternatives (Wholeben, 1980)) clearly indicates that processes which determined which schools would be closed were often devoid of clear pre stated criteria for such decisions, often ambiguous as to how the criteria which was used was finally analyzed to produce the lists of schools for closure, and sometimes absented both valid content and reliable process from the decisionmaking process. The most blatant episode reported refers to the school board who divided the district into four parts, and then appointed a citizens committee representative of each area with the charge to choose their own schools for closure and submit their list to the superintendent and board for final approval.

Nearly 10 years after the educational community finally attested to the problem of declining enrollments, little has been done which would systematically analyze a school district for potential school closure targets. What is needed, but continues to be ignored, is a method for simulating the school district's evaluation and decisionmaking prerogatives (viz. expected impact of levels of school closure(s) in terms of corresponding levels of resulting program effectiveness) beyond those provided by the more generally understood

techniques of cost-benefit and cost-effectiveness analyses. Specifically, a model which could identify several alternative, operational discontinuance and continuance assignments for each school campus, and summarily choose which level of operation best fits the desires, needs and constraints of the total school district's environment, would significantly enhance the district's ability to deal effectively with problems of declining student enrollments and increasing operation and maintenance costs.

The model presented in this paper views the school district environment as one in which some number of schools must be targeted for closure as one of a dynamic nature, and where information entering the model is highly variable and must be updated regularly. In addition, any such dynamic modeling of a highly variable environment must be able to evaluate the changing feasibility of potential alternative decisions as the parameters for selecting these decisions (themselves) change. Such an environment for dynamic modeling might be generally construed as follows.



In 1978, a rather simple though comprehensive modeling approach was attempted in the initial targeting of specific elementary schools for future

closure in a school district serving a metropolitan center of approximately 180,000 people in the State of Wisconsin. The model's results identified the number of schools which were to be recommended for closure, by name, and with supporting rationale for each school's selection. In 1981, a reformulized model was applied to the needs of a small school district in the State of Washington where the superintendent had to cut not less than 213 thousand dollars from an existing district budget of 893.5 thousand dollars (24 percent reduction) with an accompanying specific rationale for each of the individual program reductions which collectively formed the recommended fiscal roll-backs.

These models, and others developed by the author, represent reformulizations of operations research techniques which typically determine product distribution routes for transportation and personnel assignment strategies for product service delivery. The technical models themselves, commonly referred to as binary (1,0) integer programming formulations, are little more than systems of simultaneous linear inequalities and equations, whose dependent variables (i.e. the alternatives being evaluated and compared) take on the value of '1' for each alternative which is a member of the "solution set of alternatives", and the value of '0' otherwise. Constraints to this decisional process are represented in the form of linear inequalities and equations, and the single, best solution then chosen from the range of feasible, alternative solutions through the deployment of a single vector sum known as the objective function value.

TYPE OF MODEL

The technical description of the modeling formulation for determining which school sites will be targeted to remain open or be closed, and the extent of that closure, must include components of simulation modeling related to the model's ability to be hierarchical, fixed level, exclusive, exhaustive and simultaneous.

Hierarchical Decision Modeling. A hierarchical decision model is one where the alternatives (range of potential decisions) represent levels of a decision, such as might be reflected in the questions:

"How many schools should be closed, which ones, and therefore which schools should remain operational?"

"What should be the disposition of those schools targeted for closure?"

"Should there be any operational changes to the schools which remain open, and if so, what variance is warranted?"

Thus, and as will be discussed in the next section, the alternatives available to the model for determining possible school closures represent a comprehensive approach to a very complex problem. In the hierarchical school closure model presented in this paper, each of these three levels of decisioning are adequately simulated and evaluated.

Fixed Level Options. Hierarchical modeling structures may be either fixed or variable level. Fixed level means that the lowest level in the alternative decisional hierarchy available for evaluation and comparison is the same in number and type for each of the alternatives identified in the highest level of the hierarchy. The model which identifies schools for future closure is a fixed level model, in that the same five (5) lower-level hierarchical alternatives (closure with sale, closure with lease, closure with mothballing, partitioning with lease, and continued full operation) are available for each of the school sites being modeled.

In contrast, the budget reduction model is variable-level hierarchical, and as such allows different numbers and/or types of lower-level hierarchical alternatives across each of the higher-order alternatives. This represents the use of a graduated reduction scheme, in that some budgets may have several levels of percentage reductions being considered, while other have only a few. Some programs may be evaluated for potential increase in funding, while others have the simple alternative of remaining solvent at current funding levels (called zero-budgeting or stasis) or terminating their operations (100 percent funding reduction) in the future.

Discrete Options as Mutually Exclusive. In the hierarchical school closure model for school districts, two internal controls must be effectively implemented through model design.

First, each alternative option must be designed so as to represent a mutually exclusive relationship with each of the other options. Thus, if option 'a' includes some or all of the ingredients of another option, say 'b', then 'a' must be formulated with these ingredients internal to its own structure without the necessity of choosing 'b' in consort with 'a' if a combination of such ingredients are desired in the final solution. Furthermore, the characteristics of 'a' must not be equivalent to 'b', although 'a' may contain all of 'b' in addition to other ingredients not found in 'b'.

Second, the choice of a particular lower-order hierarchical option for each higher-order alternative must be singular, or discrete. Thus, choosing alternative 'a' must necessarily preclude any subsequent selection of 'b', 'c', ... for that particular higher-order alternative. Therefore, if school 'A' has five alternatives available to it (as we will explore later) in terms of varying levels of operational discontinuance (closed) or continuance (open) -- that is, a finite of lower order alternatives -- then the decision to "close the site and sell the real property associated with the site" must necessary preclude the model's subsequent choice of an alternative which leases the property under long-term agreements.

Grouped Options as Collectively Exhaustive. The hierarchical school closure model not only controls adequately for both of these "discrete" and "exclusive" factors, but in addition guarantees that all options are equivalently evaluated and each campus site assigned a resulting level of operational discontinuance or continuance based upon data available to the model. Not only does each school have a potentially variant number of options available for evaluation, but the set of variable level options assigned to each campus is exhaustive of all potentially reasonable (and unreasonable) alternative decisions which must be considered.

Identified Solutions as Feasible, Alternative and Simultaneous. The hierarchical school closure model for district level administrators identifies (if in actuality solutions exist) all possible subsets of alternative solutions which are feasible according to the constraints placed upon the model itself. These alternative subsets of available solutions that fit the decisionmaker's

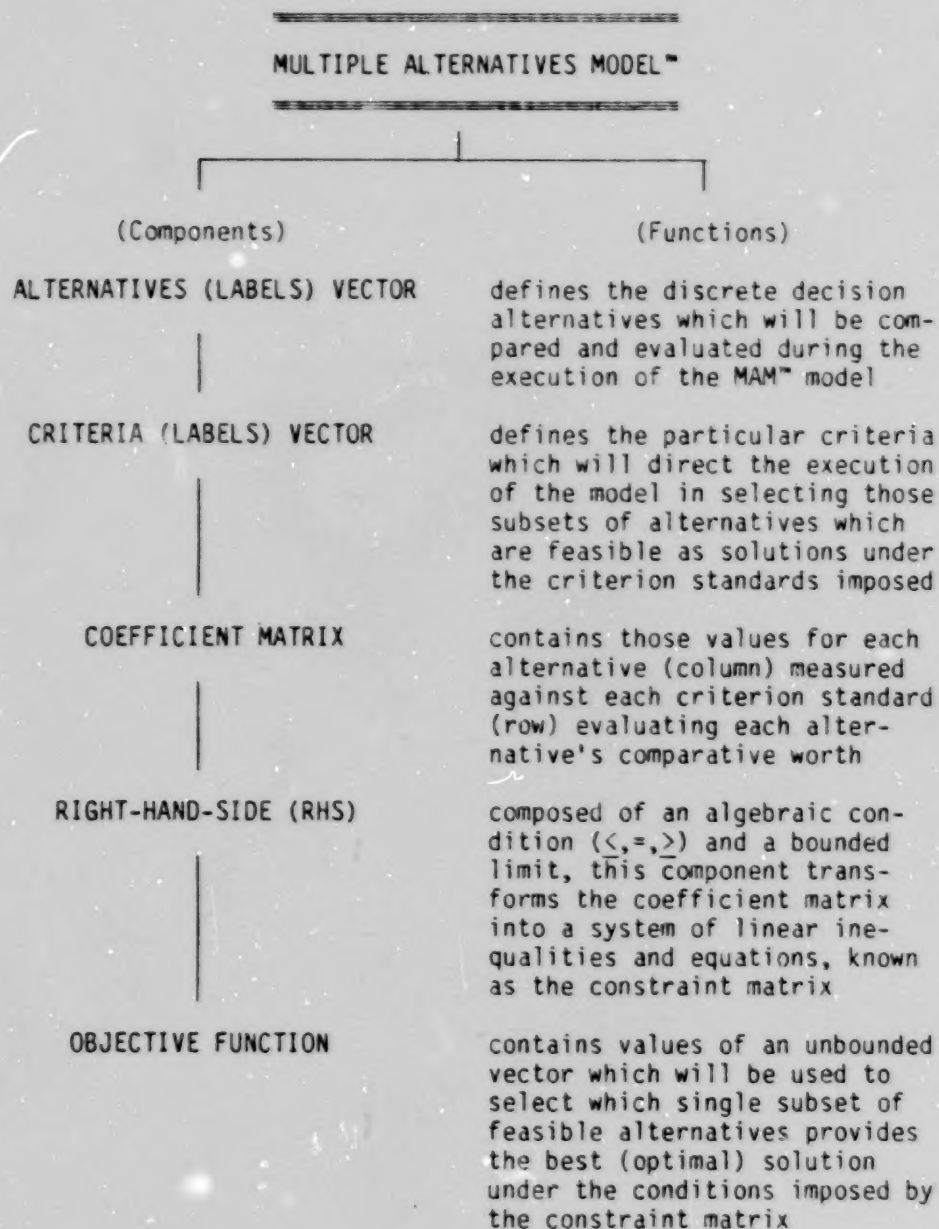
modeled constraints, i.e. are feasible, are guaranteed to not violate any of the constraints initially placed upon the model's formulation. Thus, all feasible, alternative solutions are said to be simultaneous -- simultaneously in compliance with all established modeling constraints.

COMPONENTS OF THE GENERIC MODEL

The mathematical formulation utilized for the hierarchical school closure model is that of the Multiple Alternatives Model™ (Wholeben, 1980). MAM™ is a educationally oriented revision of binary (1,0) integer programming, and represents a mathematical system of linear inequalities and equations composed of:

- [1] two (1) label definition vectors (the alternatives and criteria labels vectors);
- [2] one (1) rectangular matrix (the coefficient matrix);
- [3] two (2) column data declaration vectors (the condition vector and bounded limits vectors), collectively known as the right-hand-side or RHS;, and
- [4] one (1) row data declaration vector (the objective function).

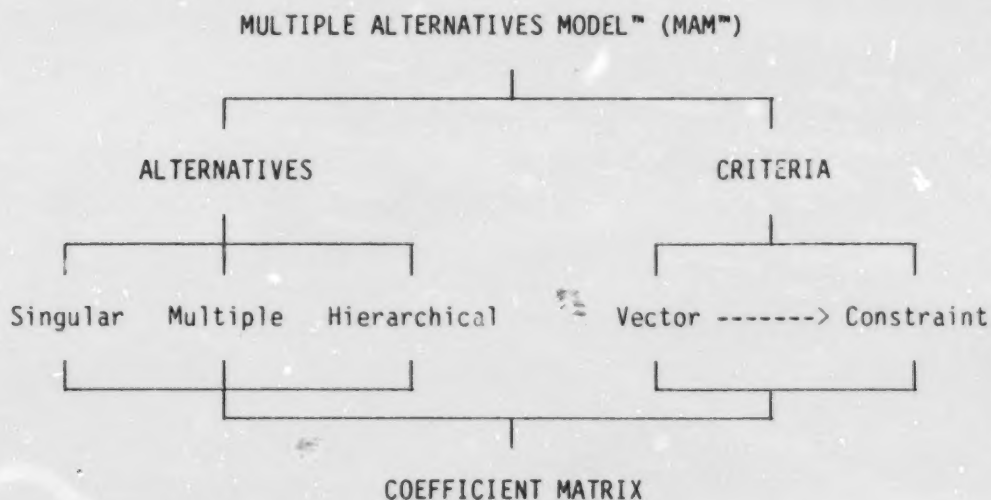
Development of the multiple alternatives modeling system is performed in a strict sequence of activities, defined by the product which results from each particular activity. This developmental sequence is described below, including a brief statement of the function of that particular product for later system definition and modeling control.



The development of the hierarchical school closure model from the more generic MAM™ framework is sequential, and involves a number of design decisions according to the mandates of the particular decisional environment being modeled. To illustrate this, each section (or phase) of MAM™ development will

be displayed in greater detail, and a number of comments related to the modeling of differential levels of school closure offered to the reader.

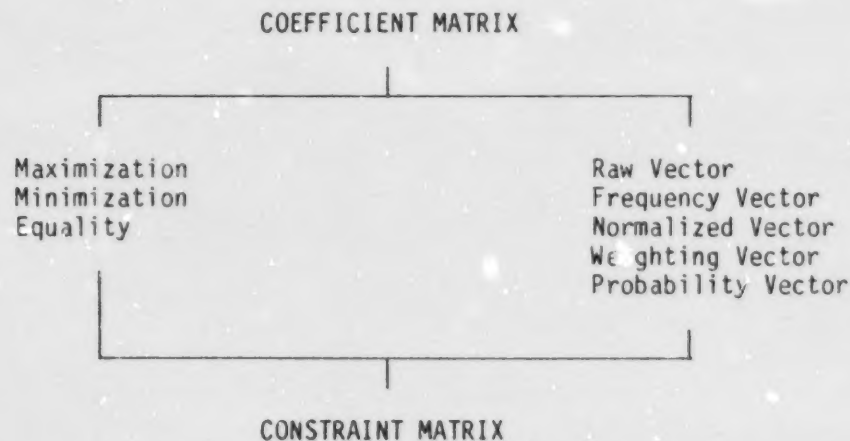
The first stage of modeling development involves the design of the alternatives and criteria (labels) vectors in advance of gathering the necessary measures for incorporation into the coefficient matrix.



The most effective and comprehensive approach to targeting existing schools for future closure requires that a hierarchical approach be taken during alternatives vector design. As is true for all MAM™ designs, the development of each criterion vector must take into account its later use as a constraint (algebraic inequality or equality) for the evaluation, comparison and selection process.

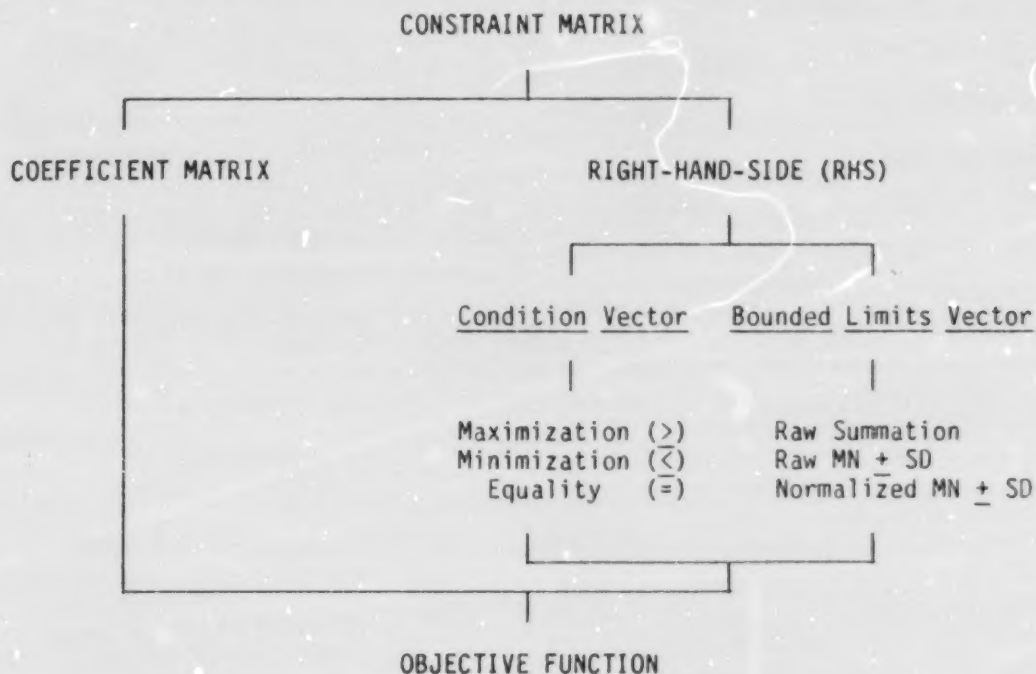
Design and construction of the coefficient matrix requires the developer to first choose what type of vector/constraint is to be modeled; for example, a maximizing vector which will be utilized as a maximizer constraint, attempting to secure that mix of alternatives which maximizes some perceived good or benefit, as opposed to a minimizing vector which will be utilized as a minimizer constraint, attempting to secure that mix of alternatives which minimizes some perceived negative effect or injurious worth. Secondly, the developer must select what type, format and scaling of values for each vector will be utilized

to display the positive or negative impact associated with each alternative's worth.



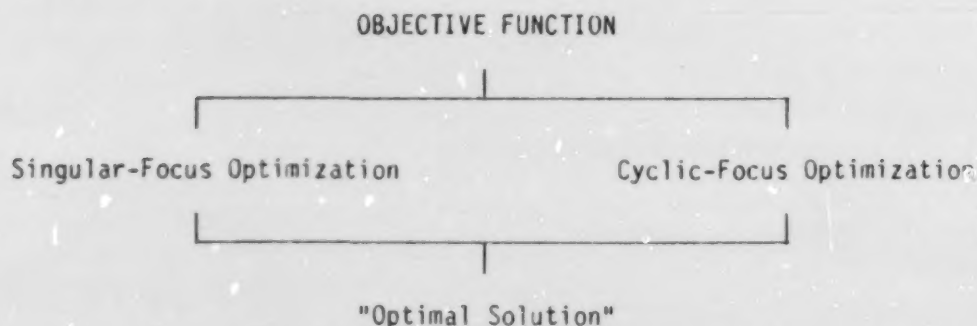
The hierarchical school closure model utilizes all three types of constraint conditions ($\leq, =, \geq$) and may employ each of the five measurement formats (raw, normal, etc.) available to the model based upon designer needs. Normalized and frequency (counting) vectors are the predominant choice for most budgeting situations. All four of the standard measurement scales (nominal, ordinal, interval and ratio) can be reliably introduced into the model.

The constraint matrix is simply a merger of the coefficient matrix and the right-hand-side (RHS) composite (condition vector and bounds vector), forming the system of linear inequalities and equalities which will constrain the final selection of the alternatives for membership within the solution set. Most problems associated with the proper use of the constraint matrix arise from an inappropriate development of the bounded values vector within the right-hand-side.



For the hierarchical school closure model, the use of normalized values for the coefficient matrix and the RHS is strongly recommended. Some criterion foci will have both raw and normalized vectors reflecting their influence upon the model, since raw values will control for specific criterion impact while the normalized transformations of these raw values will control for relative impact compared against the distributional qualities collectively of that particular criterion's measures.

The final component of the Multiple Alternatives Modeling™ framework for constrained decision selection is the objective function, or optimality vector. The objective function is either a raw or normalized values vector which is used to select the best (viz. optimal) subset of alternative solutions, from the larger subset of all feasible subsets of alternative solutions generated by the model. The use of a single objective function is known as singular-focused optimization, and is the conventional approach. Using several objective functions, and studying the impact upon the system by way of the different solutions selected by the model for each objective function used, is known as cyclic-focused optimization, and is the preferred approach.



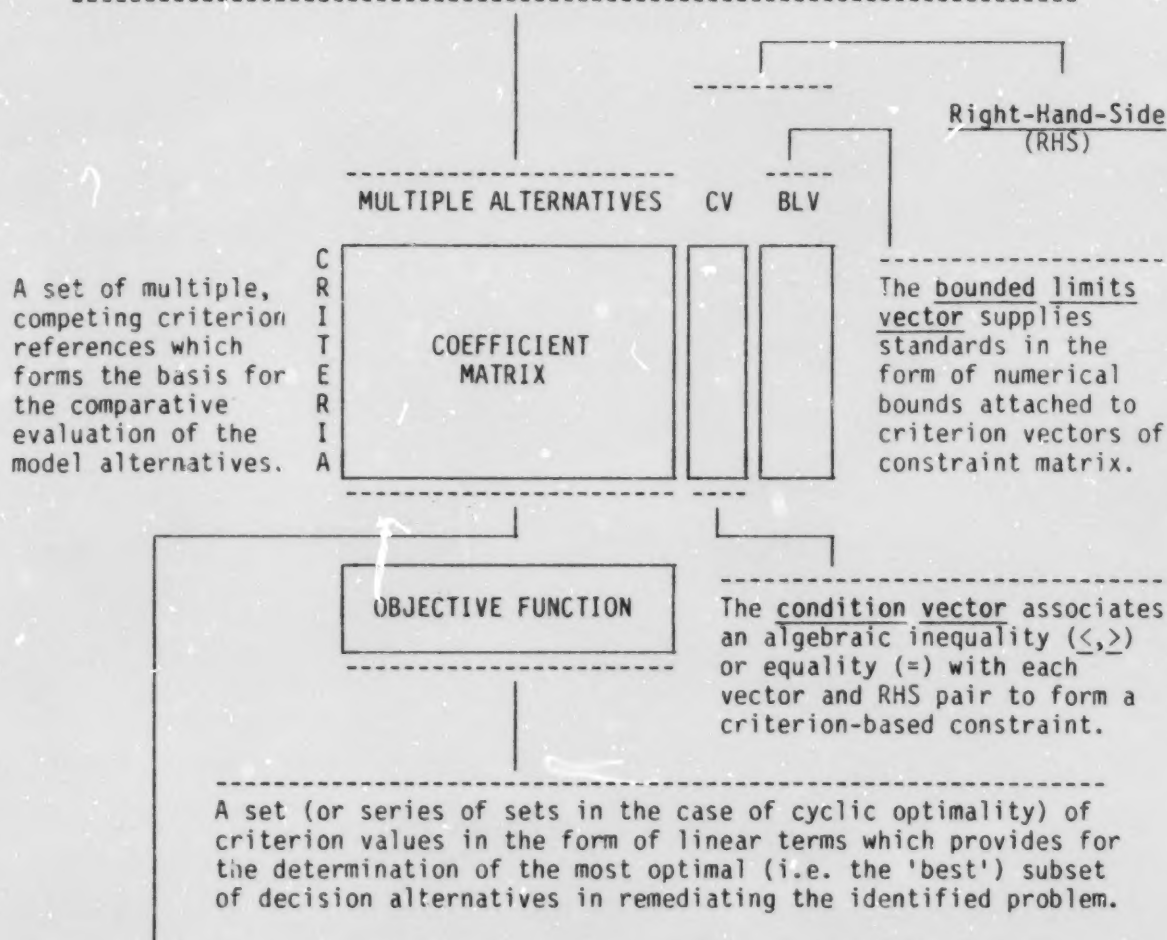
The hierarchical school closure model employs the cyclic approach, and employs a wide range of both parametric and nonparametric statistics to perform post-hoc analyses on the variance in alternative solutions observed.

In addition to cyclical optimization, the hierarchical school closure model also utilizes a step-wise alternative decision selection technique which will be explored in greater detail in a later section of this paper.

In summary, the general Multiple Alternatives Modeling™ framework employs a number of discrete vectors and a single rectangular matrix to operationalize the environment required for hierarchical decisionmaking in the targeting of individual school campuses for potential future closure.

As a total system, the MAM™ may be illustrated as follows.

A collectively exhaustive set of mutually exclusive decisions which have been designated as comprehensively reasonable reactions to the problem situation identified; and from which a subset of decision alternatives may be drawn to construct the final solution set.



The vector portion (coefficients of the left-hand-side) which denote for each of the modeled alternatives (viz., the dependent variables), the value of each particular criterion's measure or influence (viz., the independent variables) related to that same particular alternative. Thus, the coefficients of each term in the linear inequalities represent the specific criterion measure for that particular term (i.e. alternative).

Recall that the use of the coefficient matrix in conjunction with the right-hand-side (RHS) provides the basis for formulating individual subsets of alternatives designated as feasible solutions (based upon the criterion constraints

established through the simultaneous linear inequalities). These subsets of feasible solutions are then analyzed via the objective function(s) for the final determination of the single optimal solution subset.

To enhance the reader's understanding of not only the use of the MAM™ as a highly sensitive and specialized approach to hierarchical school closure modeling, but also for viewing the MAM™ framework as a generalized technique for any situation which is data-based and includes discretely defined alternatives, subsequent sections of this paper will note those attributes of the hierarchical school closure MAM™ formulation which represent marked deviations from the conventional MAM™ formulation as well as why such deviations were necessary.

STRUCTURE OF THE ALTERNATIVES VECTOR

The alternatives vector is a nominal vector, defining the hierarchical structure of the model, and naming each of the dependent variables (alternative dispositions of the school site if closed, or type(s) of continuance if left open and operational) available for evaluation and comparison. For the hierarchical school closure model, the structure of the alternatives vector is typically of two indices, the first naming the school being modeled, and the second defining the alternative dispositions of the campus based upon either targeted closure or targeted continuance of operations. Since this model is fixed-level hierarchical, the lower-order options will be identical in both number and type.

Alternative-j (j=1,...,n) ==> School Campus/Site				
Option-1	Option-2	Option-3	Option-4	Option-5

- where:
- Option-1 = target the site for future closure, and prepare the buildings for sale and relinquish of control;
 - Option-2 = target the site for future closure, and prepare the buildings for lease under district control;
 - Option-3 = target the site for future closure, and prepare the buildings of 'mothballing';
 - Option-4 = target the site for functional partitioning with partial physical units of the campus for lease; and,
 - Option-5 = target the site for continued full operation as an instructional center.

The determination of both the number and type of lower-order decisional alternatives in the hierarchical school closure model is based upon historical responses to the need to discontinue instructional operations at one or more school sites in districts such as Madison, Wisconsin and Seattle, Washington. The number and type of options advisable for any particular district could, and should, change the above alternatives vector to suit such individual needs. This model should never be operationalized before an extensive needs assessment within which the parameters of any potential closure decision(s) will be better understood. To better understand these five options, each will be explored in greater detail.

Closure and Sale. Depending upon the size and location of the school district, the option of selling a school after its closure has been one of the more typical alternatives. Many districts have neither the means nor the community demand to allow a creative use of a closed school plant. Therefore, the conventional approach is to attempt to recoup some amount of monies on their real property investment over the several preceding years, and use the monies from the sale of the plant to finance other competitive district needs. For more industrialized communities, and areas of population concentration where businesses and/or community service organizations vie for limited space on a

cost-effective basis, a closed school (depending upon its physical condition) can be a very good investment.

Closure and Lease. In several instances, the district might foresee a future resurgence in public demand for the site as an instructional center (e.g. a future rise in enrollment over the short-run). Therefore the district is well advised to not relinquish control of the site. Schools are very expensive to build, and temporary discontinuance of operations over the short-term will prove beneficial if future needs for the site are predictable. In other situations, the district may gain school board approval to act in behalf of the district as a real property manager, making portions of the closed site available to local business concerns, service organizations, etc. under terms of fixed or variable term leasing. In this way, the district can build a marginal self-sufficiency of fiscal resources wherein leasing revenues can be used to support other district projects more instructional in their orientation.

Closure and Mothball. Whether predictable reopening for instruction in the future, ill-advised terms for available leasing agreements, or simply the non-availability of individuals for leasing or purchase of the site, the school plant can not be left in a state of inactivity for extended periods of time. Buildings deteriorate from lack of use -- dust, moisture, and temperature all militate to reduce the physical quality of a building more rapidly than if the school was over populated with students. The technique known as 'mothballing' prepares a building for an extended interim pause in operational use, but is not without its own expenses. The mothballing of a school plant is typically resorted to when the future use of the building can be foreseen in a matter of a few years. The most innovative mothballing techniques, with live-in caretakers, took place in Seattle, Washington in the early 1980s.

Functional Partitioning and Lease. A very utilitarian approach to school closures during periods of declining enrollments can be found in the redistribution of students by grade-level to different plant sites. For example, a district which might have generally configured its elementary and secondary schools as K-6 (elementary), 7-8 (middle) and 9-12 (high) may find it advisable to reconfigure (i.e. redistribute) the grade-level system as a K-3 (primary),

4-6 (elementary), 7-9 (junior) and 10-12 (senior) instructional operation, and on different school campus sites. Moving from a K-6 to a K-3 orientation, for illustration, might free as much as 60 percent of the plant for purposes of sub-site leasing to community service groups, etc.. This is known as functional partitioning of the instructional plant. A number of inherent problems are associated with this alternative, not the least of which are supervision, control and security. However, functional partitioning and sub-site leasing is a most viable alternative to full-site closure.

Full Operational Continuance. Finally, of course, there is the occasion of the school plant being viable enough to sustain full instructional operation without the intervention strategies suggested in the previous four options.

An expanded view of the alternatives vector for the hierarchical school closure model is seen as follows:

Alternative (j)					Alternative (j+1)				
Opt-1	Opt-2	Opt-3	Opt-4	Opt-5	Opt-1	Opt-2	Opt-3	Opt-4	Opt-5

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--> full instructional operation continuance
--> functional partitioning with sub-site leasing
--> total closure with mothballing under district control
--> total closure with full site leasing under district control
--> total closure with district relinquishment and sale of campus site

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In the original school closure (non-hierarchical) model, only two (2) options were evaluation modeled -- target the site for closure, or allow the school to remain operational -- and therefore only two choices available to the decisionmaker. The newer hierarchical model allows greater flexibility on the part of the district superintendent when declining enrollments and/or dwindling

fiscal resources signal a need for austerity operations in terms of the physical school plant.

THE PROGRAM OPTIONS (SITE CLOSURE ALTERNATIVES) SCENARIO

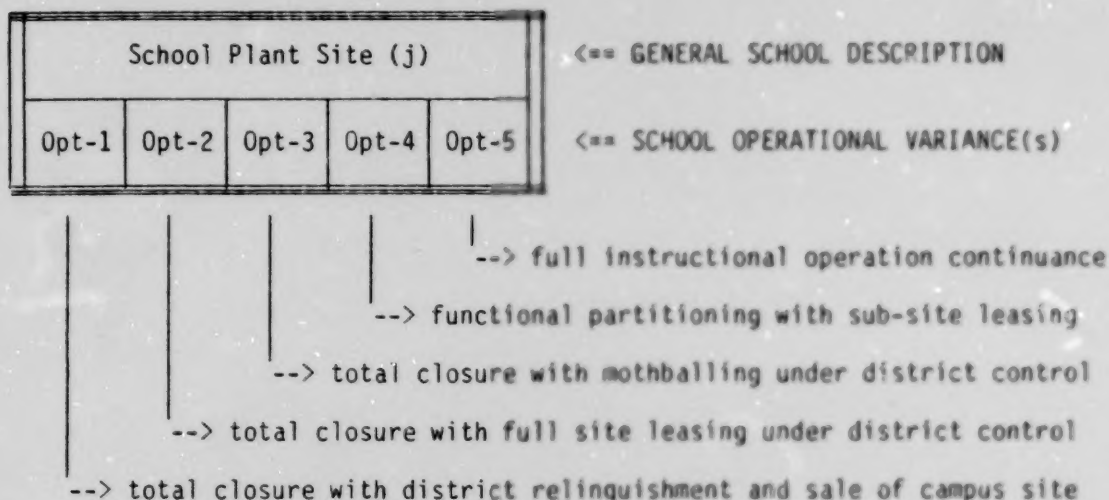
Although the program options scenario is not a "technical component" of the model itself, the development of a scenario or narrative for each of the lower-order closure decision options should be viewed as extremely important. Since it is highly unlikely that a model could be effectively designed in the absence of knowledge contained in the components of a scenario itself, the development of such a document is simply the transformation into writing of what the alternatives vector represents to the model builder.

The program options (or site closure alternatives) scenario is a narrative-based document which explains each of the alternative (lower-order hierarchical) operational continuance/discontinuance options which will be evaluated for each of the alternative (higher-order) school campus sites. Therefore, each school being evaluated will have five (5) narratives, one for each of the five possible closure options available, explaining what such an option will mean for the school, its surrounding community, and the district as a whole.

The narrative should be construed as having two general parts. The first section, or general school description section, should declare the intended mission and supporting goals (including objectives and activities) of the school. Conventionally, this description expresses the historical context of the plant, its instructional role (especially if there exist instructional ingredients both native and unique to the campus), and its relative ability to continue the instructional mission in the future.

The second section, or school operational variance section, discusses the positive and/or negative influences (e.g. effectiveness, efficiency, satisfaction, economy, etc.) linked to the specific alternatives of variable closure or continuance associated with that lower-order hierarchical alternative. Typically, this section will discuss the impact to the community and the school district's service mission, and therefore the school's intended services, which

the particular continuance/discontinuance alternative would herald. In most cases, this is best done through the direct identification of goals (or objectives, etc.) which would be beneficially enhanced or deleteriously effected due to the proposed type/level of closure being suggested.



The reader should recognize that such a narrative description (or scenario) should not be difficult, since the school campus had already been studied to the extent that certain changes (e.g. closure/sale v. closure/mothball) could be considered competitively viable. Therefore, the 'options' scenarios are a sine qua non part of the model albeit less-than-technical component of the mathematical system itself.

STRUCTURE OF THE CRITERIA VECTOR

The use of the term 'criteria vector' can cause some initial confusion, as the model builder attempts to discriminate between the column vector of criteria labels which define the rows of the coefficient matrix, and the actual rows of the coefficient matrix, themselves row vectors of measures which associate the value for each particular criterion to each specific alternative (column of the coefficient matrix). Normally, this confusion is more distracting than consequential.

The criteria vector defines (1) the type or focus of each criterion which is to be used to control the evaluation, comparison and selection from among all available alternatives, and (2) the specific row of the coefficient matrix which will contain these criterion values, important for the proper matching of the corresponding condition(s) and bound(s) in the RHS (right-hand-side). We will attempt in this section of the paper to illuminate the purpose and structure of the criterion vector as a vector of values (worth, detriment, etc.) which will ultimately be used to constrain the selection of a subset of alternatives, and their subsequent identification as feasible and/or optimal, from the existing set of alternatives in the alternatives vector.

Overview of the Criterion Reference. The worth of any criterion lies in its ability to provide direction to the selection of likely alternative actions which are both (1) feasible as potential solutions and then (2) cross-comparative (between feasible alternatives) so that the best (optimal) alternative can be identified and selected as the final solution. In the hierarchical school closure model, all criteria are necessarily expressed as numerical quantities. This does not preclude the use of non-numerical categories of criteria, since numbers can also define nominal measures (classification variables) for introduction into the model.

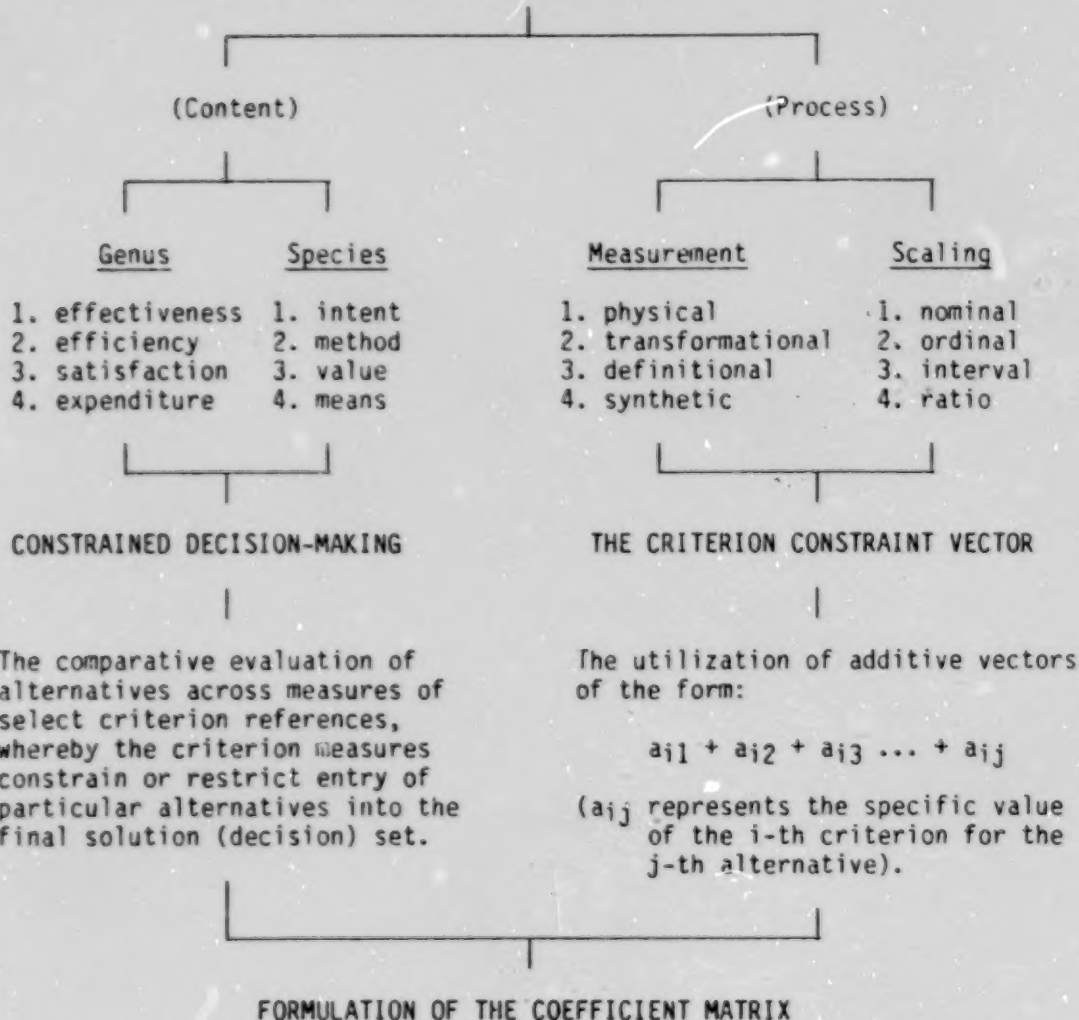
It is beyond the scope of this paper to discuss in detail the technical aspects of measurement associated with criterion vector identification, design, development and construction. However, without such information the reader's grasp of the multiple alternatives™ method in determining whether a campus will be targeted for closure or not, and if so what will be the disposition of that real property on that campus, may be significantly precluded. Therefore, we will explore this subject briefly as an attempt to optimize what should be understood versus what should be accomplished in a paper of this length.

Consider the following diagram.

THE CRITERION REFERENCE

A quantitative measure of direction and extent which, for each identified decision alternative, provides a value of genus and species origin to each of the defined comparative standards for the evaluation of multiple alternatives.

CONSIDERATIONS OF CRITERION DEFINITION



Basically, this diagram represents the criterion vector as the row-wise component of the coefficient matrix, whose sole function is to provide values upon which the model can constrain (i.e. direct and control) the comparative evaluation of the available alternatives. What is important to note is that the measures comprising the vectors may be physical (directly able to be measured by a reliable device or means), transformational (a ratio or composite of physical measures), definitional (a concoction of physical measures which purports to value some criterion not directly measurable by conventional means), and synthetic (the introduction of data of variable reliability, e.g. survey data) into the model. These four types of data may be entered into the model with numerical scales of either nominal, ordinal, interval or ratio gradient.

Whatever the type of data, and its supporting measurement scale, the vector of criterion values must be additive,

$$a_{i1} + a_{i2} + a_{i3} + \dots + a_{ij} \rightarrow \text{sum } (a_i \text{ across values } j=1, \dots, n)$$

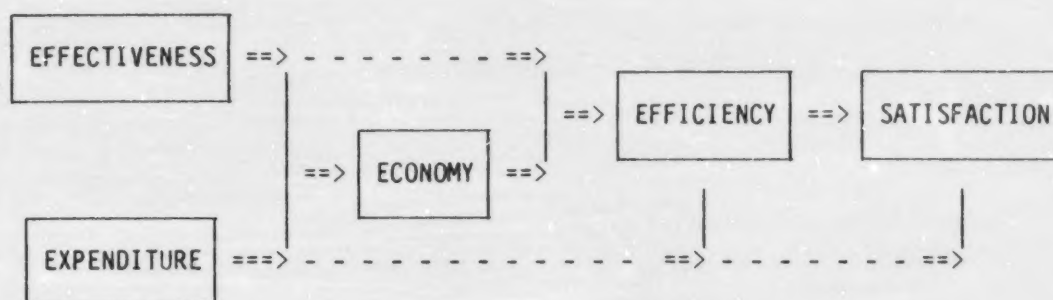
i.e. have the ability to produce a sum which represents a composite value of several alternatives (the sum of their criterion values) interacting within the model as a whole. The ability to produce an additive composite measure is all important to the subsequent ability of the model to produce a solution which is reliable based upon the model's design.

Measures of Generic Focus. The development of any criterion reference should begin with a description concerning, in general, 'what' is sought as a criterion, and 'why'. From such a broad base, more specific measures can later be identified which would support such generic foci. Typically, such generic foci will fall into one (or more) of the following general categories: effectiveness, efficiency, expenditure, economy and satisfaction. For the reader's acknowledgement, we will define these generic foci briefly, offering a diagrammatic view as to how each category relates to the other four.

Measures of 'effectiveness' typically portray an alternative's ability to comply with existing standards and policy requirements, while parallel measures of 'expenditure' demonstrate the costs associated with these levels of effec-

tiveness. Measures of 'economy' define the alternative's demand for resources, differentially displaying the variable levels of effectiveness which will result from variable resource outlays. Measures of 'efficiency' are corollary to those of 'economy', concerned more with program effectiveness than with expenditure, and defines an alternative's comparative ability to produce contrasting levels of results based upon differential resource outlays. Finally, measures of 'satisfaction' portray an alternatives's level of attraction to the managers, contributors and users of the system wherein the alternative will play a role.

Graphically, the relationships between these five (5) generic categories of criteria might be viewed as follows:



With this in mind, we will now explore more specifically some of the typical criterion references which might be incorporated into a hierarchical school closure model for program-based resource allocation in higher education.

Measures of Estimated Expenditure or Economy. To perform evaluation modeling and decisionmaking related to the targeting of school sites for either closure or operational continuance, and the level of that dicontinuance or continuance, respectively, the model must have access to cost data related to each of the various program options being compared and contrasted. Typically the costs related to each option will reflect required funding levels for each program, not the amount saved if a particular option is chosen. Such a convention allows the modeler to address increase alternatives the same way as decrease alternatives. In addition, costs related to any program can be categorized by type, account and/or responsibility. Therefore, several criteria

related to costs should be modeled, each relating to a different type (account, etc.) in order to discriminate adequately between variable cost categories. Such breakdowns allow the decisionmaker to later restrict (or relax) select, individual cost criteria without indirectly affecting other categories of expenditure.

Expenditure criteria should be modeled in both raw (e.g. dollars) as well as normalized (e.g. T-scores) data formats. As mentioned previously, raw formats allow the model to specifically control for actual dollar-expenditures which is necessary when the model must produce solutions which do not surpass the anticipated total budget of the university (or college, etc.). Normalized formats help the model control for 'relative to central tendency' comparisons. With normalized measures, the comparison between alternatives are expressed in deviations from the mean regardless of the actual high (or low) cost of the programs being evaluated.

Measures of Historical and/or Forecasted Effectiveness. The appropriate evaluation, comparison and selection of hierarchical school closure alternatives can not be accomplished without addressing the past, current and future effectiveness of each program (e.g. within the school) option. Obviously, high cost programs if also superiorly effective should be perceived by the model as a relative benefit to the organization, in contrast to low cost programs which might be marginally effective or even progressively obsolescent. Effectiveness can be measured in many ways, depending upon the particular goals and objectives of the program. Enrollment trends, accreditation reports, and faculty reputations might all be viewed as acceptable measures of program effectiveness.

Effectiveness measures should also be introduced within the model in both raw and normalized formats, although normalized measures are more often the rule. An interesting case in point is when the model builder does not wish to express a raw standard to distinguish between 'what is effective' and 'what is ineffective', but rather compares measures for their relative deviation from the 'average effectiveness measured across all programs being compared'.

Measures of Comparative (Pairwise) Efficiency and/or Satisfaction. Measures related to program efficiency and satisfaction will typically involve pairwise

combinations among available alternatives. Such an approach may be necessary because the determination of 'what is efficient' and 'what is satisfying' might only be possible when the question 'as compared to what' is likewise made. Other criteria (e.g. effectiveness) do not often fall into this pairwise categorization since policy standards typically state 'what is to be considered effective' (or at least organizational policy should).

Do not fall into the trap that presupposes an increase in expenditure automatically results in a corresponding increase in efficiency, or alternatively that decreased funding can stabilize effectiveness through austerity actions of increased efficiency.

Measures of efficiency and satisfaction almost always involve surveying an individual's perceptions or feelings, and accepting the problems inherent in the varying reliability associated with these measures. Such data is typically known as synthetic, and will be dealt with in the following section.

Measures of Differential (Population-based) Perception. Measures of perception (or feelings) can (and should) be utilized in the hierarchical school closure model, although the modeler must control as much as possible for the unreliability characteristics of such 'tell me what you think' measurements. The modeler must recognize that the model does not differentiate between data based upon the extent of the data's reliability, but rather treats the data as if it were perfectly reliable (and valid) in every way.

A second issue which must be addressed when modeling perceptual data is that of additivity, that is, the ability of individual data elements to be added and produce a sum which is meaningful as a composite measure. This issue will be addressed in greater detail in a later part of this paper.

Measures of Internal Modeling Control. Like many decision simulation models which are hierarchical and fixed level, the model itself does not automatically discriminate between the various 'close-open' variance options within one school and similar options within another unrelated school. In addition, the model must be specifically 'told' that one and only one closure option must be

selected as a solution for each particular campus site, and furthermore that no site should be excluded during this process. What may seem like a formidable task to the novice programmer is actually the easiest part of criterion development during modeling formulation.

Although the modeling of these internal decisional control elements will be discussed later in this paper, the reader can view such criteria as follows. Each school will be evaluated across five options. To assure that each school is so evaluated, and that one and only one of the available options are considered viable, a criterion vector of unary elements (viz. 1's) will be supplied to the model for each of the school under evaluation. Another vector might link certain lower-level options of different schools together in order to compare certain sites which have something in common. Therefore, these control-criterion vectors are simply composed of the element '1' and posited in the vector wherever an alternative must be addressed by the model, and therefore wherever other alternatives are to specifically ignored.

Such a control vector would appear as follows:

	opt1	opt2	opt3	opt4	opt5	
	----	----	----	----	----	
(control criterion)	+1	+1	+1	+1	+1	= +1

informing the model that all five options are to be considered, but one and only one alternative must be selected as the final solution.

Criterion Constructs for Targeting School Closures. To illustrate the types of criterion foci which play a valuable role in the targeting and selection of school sites for potential closure, a listing of general constructs follows.

CONSTRUCT #1: THE PHYSICAL PLANT

[SIZE OF FIRM OR BRANCH]

size of firm or branch relative to current consumer demand

Illustration:

capacity of educational site and the current/potential enrollment trend(s) in the attendance area

[TYPE OF CONSTRUCTION]

relationship of physical construction to other existing criterion variables

Illustration:

characteristics of plant construction in relation to remaining criterion estimates

[AGE/CONDITION OF SITE]

relation of plant age and current condition of facility to other criterion estimates and on-going operations

Illustration:

impact of school condition and age upon facility costs and academic requirements

[COST OF SITE AND PREMISES]

property costs associated with purchasing and/or leasing current site/premises; and subsequent maintenance of site for future production

Illustration:

current principal and interest costs associated with school plant

[TAXES]

loss to profit margin due to existing regional tax structure

Illustration:

increase to overall budgetary requirements due to tax requirements

[UTILITY RESOURCE REQUIREMENTS]

demand upon natural and converted resources for production and operation

Illustration:

requirements of heating and lighting utilities for facility operation

[UTILITY RESOURCE COST EXPENDITURES]

current fiscal expenditures for resource resource requirements

Illustration:

current fiscal expenditures for utility requirements

[ROOM FOR EXPANSION]

feasibility of site expansion due to increased product demand

Illustration:

amount of 'non-facility' grounds available for construction additions without off setting existing playground area(s)

[SECURITY RISK]

characteristics of plant and surrounding area promoting security of facility

Illustration:

degree of school complex security fostered by physical construction and neighborhood location

CONSTRUCT #2: THE LOCATION OF THE FACILITY

[INDUSTRIAL CONCENTRATION]

present and/or proposed concentration of parallel competitive activities within the marketing area under consideration

Illustration:

ratio of existing educational outlets (schools) to the localized supply of available school-aged children

[TRANSPORTATION DISTANCE]

transportation distance of goods and services between production site(s) and various supply and distribution centers

Illustration:

transportation distance of students within assigned attendance zone to respective school site

[TRANSPORTATION COSTS]

incurred costs of transporting goods from production centers to distribution outlets

Illustration:

expenses associated with required busing (e.g. special education, desegregation, etc.) and remote-residence transportation of students (residences beyond established walking-distance regulations)

[REMOTE OR RURAL LOCATIONS]

effect of dependence upon required utility facilities not immediately available to a remote production center

Illustration:

effect of skewed student-residence (location) distributions upon centralized school site location within perimeter attendance areas

CONSTRUCT #3: THE NEIGHBORHOOD

[RESIDENTIAL HOUSING AVAILABILITY]

potential residential construction and effect upon consumer characteristics

Illustration:

potential residential construction and impact upon future school-aged populations

[AREA INDUSTRIES]

compatible local industries as positively influencing the growth (or stable margin) of consumer magnitude

Illustration:

availability of local industries promoting family residential magnitudes (e.g. fire and police protection, groceries, churches)

[RELATED EMPLOYMENT OPPORTUNITIES]

availability of neighboring industries (business, industry, schools) to provide additional income possibilities for dual employment families

Illustration:

availability of employment opportunities related spouse occupational interests

[AUXILIARY FACILITIES]

availability of related secondary interests to consumer population (e.g. deprived area regarding new employment source)

Illustration:

Extent of school facility utilization for extra-educational affairs (e.g. community meetings, recreation center)

[ENVIRONMENTAL IMPACT]

impact of plant site and production operations upon surrounding area (e.g. quality air control, extent of vehicular traffic, property values, tax bases)

Illustration:

relation of neighborhood school to surrounding residential area (e.g. property values and tax base, residential construction, relocation of supporting industries)

[INTER-INDUSTRIAL PLANNING INDICE]

extent to which surrounding non-competitive industry planning foci match current research and development data

Illustration:

use of community and regional (city, county, state) planning data to assist educational decisioning

CONSTRUCT #4: THE FACULTY/EMPLOYEE POPULATION

[ENTREPRENEURIAL AND EXECUTIVE PERSONAL PREFERENCE]

management-based objectives in the design, production and supply of separately, manufactured goods

Illustration:

curricular and instructional demand of school administration related to professional biases

[MANAGEMENT AND TECHNICAL STAFF]

availability of and present average costs for management associated with desired production levels

Illustration:

characteristics and salaries of school administrators associated with instructional foci and parental demands

[LABOR]

availability and present average costs of labor associated with desired production levels

Illustration:

characteristics and salaries of teacher staffing and emphases associated with instructional foci and parental demands

[PRODUCTION COST COMPETITION]

relationship of resource requirements to production output

Illustration:

allocation of budgetary requirements related to class size, teacher load, and curricular attainment

[EDUCATIONAL FACILITIES]

availability of external training and inservice resources for improving production and supply

Illustration:

availability of higher education facilities for improving staff knowledge and instructional techniques

CONSTRUCT #5: THE CONSUMER/STUDENT POPULATION

[PROXIMITY OF RAW MATERIAL SUPPLIES]

accessibility of raw material and supplies to manufacturing location

Illustration:

proximity of students within walking distance of school site

[BUSINESS GROWTH]

current business trends in consumer population and related demand potential

Illustration:

projected student enrollment based upon recent attendance trends

[CONSUMER DEMOGRAPHIC CHARACTERISTICS]

consumer demographic characteristics for future trend forecasting

Illustration:

trends of family size, etc., for predicting future enrollment potential

[AFFIRMATIVE ACTION]

current (and proposed) level of federal guidelines' compliance

Illustration:

current perspective relating to desegregation and minorities (including handicapped, special education) commitments

CONSTRUCT #6: THE PRODUCT MARKET (CURRICULUM & INSTRUCTION)

[MARKET PULL AND MARKETING STRATEGIES]

consumer demand (market pull) and manufacturer's planned focus of supply (marketing strategy)

Illustration:

curricular demand emphases (basics curricula, individualized instruction, special education) of parents in attendance area, and the current (and potential) availability of such academic activities within the school

[LABOR STABILITY]

turnover history and/or projected availability of technical staff in production (labor, etc.)

Illustration:

trend characteristics of instruction staff parallel to established instructional objectives

[FACILITY UTILIZATION]

extent of current production utilization of existing plant resources

Illustration:

degree of academic and co-curricular utilization of existing school facility resources (e.g. class space, specialized instructional areas)

[PRODUCTION INDEX]

current rate of production output based upon existing resources and/or facilities

Illustration:

level of academic attainment based upon current curricula and instructional strategies

[AUXILIARY USE OF FACILITIES]

amount of existing specialized plant equipment which is essential the mission of production

Illustration:

existence of specialized facilities integral to the academic process (science laboratory, swimming pool, built-in A.V. equipment)

[ACCESSIBILITY OF PREMISES]

availability of production site to supply/distribution centers, and of distribution centers to consumer

Illustration:

multi-dimensional curricular interests and instructional goals

To better understand this use of a single vector, and its relationship to the other criterion vectors which will ultimately and simultaneously constrain the final selection of the 'total solution', we will now explore the next component of the multiple alternatives modeling™ framework, the design and development of the coefficient matrix.

STRUCTURE OF THE COEFFICIENT MATRIX

The function of the coefficient matrix is to store the entire set of criterion values across all program options (decisional alternatives), and to make these values available to the model during execution as needed. Rows of the matrix are defined by the individual criterion focus while columns are defined by each of the options being evaluated. The coefficient matrix of criterion values across all alternative options is the only matrix in the multiple alternatives model™. This unfortunately leads to some amount of dissonance when the component called the constraint matrix is addressed.

The constraint matrix, explored in greater detail later in this paper, is simply the merger of the coefficient matrix with two column vectors known as the condition vector and the bounds vector (together called the right-hand-side). The coefficient matrix can not constrain the decisional process without the addition of these two vectors. The coefficient matrix does however contain the values for each criterion across all alternatives -- these values referred to as coefficients because of their placement in the resulting algebraic inequality or equation -- thus the rationale for the matrix's name.

Because the coefficient matrix can be partitioned into submatrices (i.e. parts of the whole matrix), and each part or submatrix named to represent the type, format or purpose of its criterion components, we will in this paper explore the function of the coefficient matrix in terms of an arbitrary partitioning of the coefficient matrix into submatrices according to data format. Partitioning is cosmetic only, and does in no way lead to a more efficient or effective model formulation as far as the model is concerned. The model developer typically partitions in order to quickly identify certain matrix parts for later data modification during the search for initial feasibility and the later need to relax or restrict certain model constraints. More on this later.

Structure of Criterion-based Data Entry. Criterion values for each closure option related to the continuance or discontinuance (levels) of a school's instructional operation(s) are entered into the coefficient matrix in much the same way as Christmas ornaments are stored in their sectioned container -- one

ornament for each section, although some sections may be missing an ornament due to breakage or its otherwise being lost. What may seem trite or trivial to the reader is hardly the case in model development. Obviously, only one value for any given criterion can reflect the worth for any given alternative. If several values are existent, then either additional criterion vectors (rows of the coefficient matrix must be constructed) or some composite measure entered into the cell of the coefficient matrix (usually the mean or the sum, depending upon the intent of the criterion and the physical measure itself) as a composite value for the criterion.

Let us view the composition of the coefficient matrix in the form of a mathematical system of appended row vectors.

		Multiple Decision Alternatives (X_j)						
Criteria (A_i)		X_1	X_2	X_3	X_4	X_5	...	X_j
<-	- Criterion A_1	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	...	a_{1j}
<-	- Criterion A_2	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	...	a_{2j}
<-	- Criterion A_3	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	...	a_{3j}
	⋮	⋮						⋮
	⋮	⋮						⋮
<-	- Criterion A_i	a_{i1}	a_{i2}	a_{i3}	a_{i4}	a_{i5}	...	a_{ij}
	----->	$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 + \dots + a_{1j}x_j$						
	----->	$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 + \dots + a_{2j}x_j$						
	----->	$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 + a_{35}x_5 + \dots + a_{3j}x_j$						
		⋮						
	----->	$a_{i1}x_1 + a_{i2}x_2 + a_{i3}x_3 + a_{i4}x_4 + a_{i5}x_5 + \dots + a_{ij}x_j$						

where:

- A_m represents each of the criterion available for evaluation modeling; $i = 1, 2, \dots, m$ criteria.
- X_n represents each of the decision alternatives to be modeled with the multiple alternatives framework; $j = 1, 2, \dots, n$ decision alternatives.
- a_{mn} represents criterion measures for each of the A-criterion across each of the X-alternatives; $i = 1, 2, \dots, m$ criteria; $j = 1, 2, \dots, n$ alternatives.
- $a_{mn}x_n$ represents each cell entry within the constraint matrix wherein the i -th criterion value associated with the j -th decision alternative is defined.

Note how each cell entry in the coefficient matrix represents the coefficients in a vector whose range of dependent variables are the alternatives (columns of the matrix) being compared for membership in the solution set. The dimensions of the coefficient matrix will always, therefore, be m -rows deep where m represents the number of criteria being modeled, and at least n -columns wide where n represents the number of alternatives being compared. Depending upon the decisional situation, the coefficient matrix might be $n+k$ columns wide where k represents the number of 'slack' dependent variables constructed to perform more complex comparisons between separate 'groups' of the dependent 'program options' variables. By extension, the dimensions of the constraint matrix will always be $n+k+2$ columns wide where $k \geq 0$ and the two (2) additional columns represent the condition and bounds vector of the RHS respectively.

The Raw Data Submatrix. Some number of rows in the coefficient matrix will contain raw values of respective criteria, and is therefore called the raw data submatrix. The raw data submatrix requires greater care in the design of the bounds vector of the RHS; therefore, this submatrix is typically small and is utilized only as absolutely necessary. Such differential use, of course, depends upon the type, format and purpose of the particular criterion being modeled. For the hierarchical school closure model, the raw data submatrix might contain measures of cost (\$\$\$ expenditure requirements), distance and time requirements related to student transportation data, measures of a plant's age and ADA (average daily attendance) capacity, and/or historical enrollments.

From the standpoint of data formats and/or types, all physical, transformational and definitional measures, in raw data form, could be contained within this particular submatrix.

The Normalized Data Submatrix. Introduced for the first time in a multiple alternatives modeling™ formulation for school closures (Wholeben, 1978), the use of normalized measures for comparing and contrasting potential decisions plays a significant role in solution membership construction for the hierarchical school closure model. Entries in the normal data submatrix are either z-score or T-score transformations of original raw data distributions, although T-scores are recommended to minimize the necessity for negative numbers and fractional entries. Normalized data allow the model to compare alternatives based upon their relative deviation from computed central tendencies in the alternatives' composite distributions. In terms of evaluation, this allows the model to compare and contrast alternatives which may not be the most desirable but which are the only options available. While this same intent could be accomplished through the raw data submatrix, the establishment of the appropriate bounds in the RHS is tedious for raw data, and oftentimes experimental at best.

This particular submatrix can contain normalized transformations of the physical, transformational and definitional raw data measures, and could if additivity is assured also contain normalized transformations of appropriate synthetic data sets. More on this later.

Contrasting the Raw and Normalized Data Submatrices. To better understand the differences between the use of raw data versus normalized measures, we will jump ahead in our discussion to material more appropriately addressed in the sections, Structure of the Right-Hand-Side (RHS) and Structure of the Constraint Matrix. We will contrast the raw and normalized data submatrices here, and review this material again for the reader in these future two sections.

The significant increase in modeling flexibility attributed to the use of normalized over raw data measures lies in (1) the model's ability to compare and contrast alternatives due to relative deviations from a distributional central tendency and (2) the standardization of bounded value assignments in the bounds

vector of the right-hand-side of the constraint matrix. As mentioned previously, normalized coefficients allow the model to select from alternatives which may not be the most desirable, and may even violate established policy, but which are the only alternatives available and represent the institution's best attempts at compliance. The use of normalized coefficients in such a situation can save literally hours of model development and testing time when compared to the raw data approach. Secondly, the choice of appropriate bounds or limits for the RHS of the constraint matrix -- those summed composites which guide the final choice of a solution from among the multiple alternatives available -- can now be a standardized procedure equivalent for all criterion vectors which employ normalized measures as their coefficients. This is easily demonstrated utilizing the following two charts.

For T-normalized scores (preferred):

 T-normals for MIN constraint (RHS => $n*(50+10)$ or $[n*60]$)
 " " MAX " (RHS => $n*(50-10)$ or $[n*40]$)

where: '50' and '10' are the mean and standard deviation
 of the T-normal distribution, respectively, and
 'n' is the number of programs being evaluated.

and, for z-normalized scores:

 z-normals for MIN constraint (RHS => $n*(0.0 + 1.0)$ or $[+n]$)
 " " MAX " (RHS => $n*(0.0 - 1.0)$ or $[-n]$)

where: '0.0' and '1.0' are the mean and standard deviation
 of the z-normal distribution, respectively, and
 'n' is the number of programs being evaluated.

 Thus, for a situation in which 200 individual programs were being compared (regardless of the number and/or variety of their lower-order options), the

bound(s) in the RHS for all criteria which are to be maximized would be either '8000' (i.e. $200 \times (50 - 10)$) for T-scores or '-200' (i.e. $200 \times (0.0 - 1.0)$) for z-scores. Alternatively, the bound(s) in the RHS for all criteria which are to be minimized would be either '12000' (i.e. $200 \times (50 + 10)$) for T-scores or '+200' (i.e. $200 \times (0.0 + 1.0)$) for z-scores. The length of one (1) standard deviation is subtracted from lower bounds and added to upper bounds simply to allow somewhat greater flexibility (relaxation) for the model's comparison and contrast of the alternatives.

The above discussion raises the interesting issue as to the determination of the RHS bounded value for an identity vector (constraint). Very simply, it is not recommended that you employ normalized identity bounds for identify vectors since it adds nothing to either the effectiveness nor efficiency of the model, whether in development or execution. Identity constraints are more typically utilized in the control submatrix, to be discussed in the next section, and employ raw data rather than normalized coefficients.

The Control Submatrix. Controls established in the control submatrix are usually of two types. The first and most typical is that of a series of '1' elements specifying which of the program options are to be evaluated and how many alternatives (in quantity) should be contained in the final solution. This is best illustrated through an example. Consider the situation where three (3) schools (higher order dependent variables) are being evaluated for a suitable level of closure (or not) across a total of five (5) close/open options (lower order dependent variables). To assure that one and only one option is selected per each of the programs being evaluated, the control submatrix would appear as follows:

A.1	A.2	A.3	A.4	A.5	B.1	B.2	B.3	B.4	3.5	C.1	C.2	C.3	C.4	C.5	RHS
1	1	1	1	1											=+1
					1	1	1	1	1						=+1
										1	1	1	1	1	=+1

where the bounds entry in the RHS would necessarily be '= +1' for each row of the coefficient (control) submatrix displayed above.

The second type of control established in the control submatrix of the encompassing coefficient matrix typically uses a raw data vector, and in the specific case of the hierarchical school closure model, the coefficients in this vector are the various school enrollments by school being modeled. The function of this raw data control vector is simply to assure that whichever schools are closed, or kept open at whether level, the total ADM (average daily membership) of the district is supported by the remaining 'open' campuses.

The Synthetic Measure(s) Submatrix. The last (typical) partitioning of the coefficient matrix (and therefore the corresponding constraint matrix) is based upon available synthetic data (e.g. from a survey of perception). Synthetic measures can be introduced into the model as interval-approximated measures if and only if the measures are additive. Depending upon the sophistication of the device which produced the measures, and the scientific stringency of the model's consumers, questionable additivity may not be a serious violation of the model's overall integrity. This is very situation based, and needs to be viewed separately based upon the context of the decisional environment.

Synthetic ordinal (and therefore nominal) measures can likewise be introduced into the model with a little ingenuity on the part of the model's designer. Consider the following illustration. A suitable group of individuals has been surveyed and asked to what extent they would agree or disagree with the declarative judgement:

"The closure of this school would deleteriously effect the district's committment towards training for learning disabled."

for each of three school sites with only three open/close strategies available, and using the following ordinal rating scale:

STRONGLY AGREE	SOMEWHAT AGREE	SOMEWHAT DISAGREE	STRONGLY DISAGREE
(1)	(2)	(3)	(4)

so that a larger number (e.g. '4') represented greater disagreement and therefore a better 'rating' for the proposed closure. In turn, such a measure would

be considered a vector where we would wish to maximize a composite sum, if in fact additivity could be assured.

Let us assume that you compute the median of the responses for each of the three options for each of the three schools,

School A			School B			School C		
0-1	0-2	0-3	0-1	0-2	0-3	0-1	0-2	0-3
---	---	---	---	---	---	---	---	---
Md ₁	Md ₂	Md ₃	Md ₄	Md ₅	Md ₆	Md ₇	Md ₈	Md ₉

and as you attempt to compute a meaningful RHS bound to compare the potential sum of median scores (1=1,...,9) a statistically oriented person states that the sum of median-based scores is meaningless and not allowable. What is a reasonable alternative?

Actually, there are several alternatives. One option is to enter a '1' for each closure option which is 'deemed acceptable' based upon an interpretation of the median computed. For example, a '1' might be entered for any option whose median was at least 3.0 or higher. Then, the sum of the '1' entries could be controlled with an appropriate (e.g. =+1) RHS-bound. A second alternative is more comprehensive and does not require such a narrow interpretation.

Consider constructing two criteria vectors, one for indicating the frequency with which the 'somewhat disagree' response was chosen for each closure option, and the other for the frequency of choosing the 'strongly disagree' response. Now this part of our synthetic submatrix might appear as:

	School A			School B			School C		
	0-1	0-2	0-3	0-1	0-2	0-3	0-1	0-2	0-3
	---	---	---	---	---	---	---	---	---
's/what disag':	27	12	56	45	23	5	34	53	2
'strong disag':	12	45	2	2	34	49	23	24	57

such that 27 'somewhat disagree' responses were recorded for School A, Option 1 as compared 12 'strongly disagree' responses, etc.. An appropriate bound for

the RHS can now be computed according to (1) central tendency, (2) predetermined limits on the 'disagreement' response, or (3) some other arbitrary condition.

We have now explored that component of the multiple alternatives model™ known as the coefficient, and out of necessity introduced you in piecemeal fashion to the use of the condition and bounds vectors. We will now discuss these two vectors in greater detail, especially as to how they might relate to the hierarchical school closure model.

STRUCTURE OF THE RIGHT-HAND-SIDE

The right-hand-side is the name given to represent collectively two column vectors which when applied to the coefficient matrix allow the multiple alternatives model™ to perform decisionmaking within a constrained environment. These two vectors, the condition and bounds (bounded limits) vectors, provide the necessary, one-to-one algebraic articulation with the vectors of the coefficient matrix to construct the system of inequalities and equalities which provide constraints to the upcoming evaluation, comparison and selection of a solution from among available alternatives.

Recall the structure of the coefficient matrix. Each row represents a vector of criterion values (coefficients) which purports to assign worth to each of the alternatives (dependent variables) available for comparison and contrast. Depending upon which of the dependent variables are being considered as potential members of the solution set, the dependent variable's corresponding coefficients are added together and compared to some bound or limit, typically a standard established by the policy of the institution. A group of dependent variables is considered feasible as potential solutions if and only if that comparison satisfies the limits established by policy.

This modeling of policy limits is accomplished through the use of the two vectors of the right-hand-side. Initially, however, each separate vector of the coefficient matrix must be defined as one of three possible criterion types: a maximizer criterion, a minimizer criterion, or an identity criterion. Recall that maximizer criteria typically model 'positive' (i.e. benefit) criteria,

minimizer criteria typically address 'negative' (i.e. detrimental) areas of concern, and that identity criteria are most often control elements for the model during execution. Consider the following diagram.

		Multiple Decision Alternatives (X _j)						
Criteria (A _i)		X ₁	X ₂	X ₃	X ₄	X ₅	...	X _j
<-	Maximizer - Criterion A ₁	a ₁₁	a ₁₂	a ₁₃	a ₁₄	a ₁₅	...	a _{1j}
	Minimizer - Criterion A ₂	a ₂₁	a ₂₂	a ₂₃	a ₂₄	a ₂₅	...	a _{2j}
	Equality - Criterion A ₃	a ₃₁	a ₃₂	a ₃₃	a ₃₄	a ₃₅	...	a _{3j}
		:	:	:	:	:	:	:
		.	.	.	.	.	.	.
		----> <u>Maximizer Constraint:</u>						
		$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 + \dots + a_{1j}x_j$						
		requires that the sum of the individual criterion cell values be equal to or greater than a specified lower bound as defined by the corresponding row entry within the conditional vector (RHS).						
		-----> <u>Minimizer Constraint:</u>						
		$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 + \dots + a_{2j}x_j$						
		requires that the sum of the individual criterion cell values be equal to or less than a specified upper bound as defined by the corresponding row entry within the conditional vector (RHS).						
		-----> <u>Identity Constraint:</u>						
		$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 + a_{35}x_5 + \dots + a_{3j}x_j$						
		requires that the sum of the individual criterion cell values be equal to a specified parameter as defined by the corresponding row entry within the conditional vector (RHS).						

After this initial definition of each criterion vector of the coefficient matrix, the model must then be informed of 'your vector-by-vector definition'. The condition and bounds vectors in the RHS perform this task very efficiently.

The Condition Vector (CV) Component of the RHS. The condition vector is a discrete column vector whose cell entries contain one of the three values: $\geq$, $\leq$, or $=$. The algebraic inequality condition ' $\geq$ ' defines the vector in the coefficient matrix as a maximizer. This simply means that a 'lower bound', defined in the corresponding entry of the bounds vector (see below), must be surpassed by the computed sum of the vector's coefficients (based upon which of the dependent variables are being considered at the time for solution membership), moreover that the final value of that sum should be as high as possible. A condition vector value of ' $\leq$ ', on the other hand, defines the associated vector of the coefficient matrix as a minimizer. This means that an 'upper bound', also found in the corresponding entry of the bounds vector, must not be surpassed by the computer sum of the appropriate vector coefficients. The level which these sum eventually reach -- high for maximizers and low for minimizers -- depends upon the level of competition existing between the various criterion constraints. A value of ' $=$ ' defines an identity vector, several examples of which have already been reviewed in this paper.

The Bounded Limits Vector (BLV) Component of the RHS. The bounds vector links with the condition vector to become what is called the right-hand-side or RHS, and which ultimately converts the coefficient matrix into a constraint matrix for alternatives comparison and contrast. A discrete column vector, the BLV has cell entries which are classified as a lower bound, upper bound or identity, depending upon the vector's definition as a maximizer, minimizer or identity, respectively. It is the sole function of the BLV to model policy limits or standards, and to provide the model with a basis for determining whether some subset of alternatives (dependent variables) are feasible (potential solutions) or not. In conjunction with the CV, the BLV establishes each vector of the coefficient matrix as either an inequality or equation. Now a system of linear inequalities and equalities, the model is prepared to begin its sorting and culling of the available alternatives.

It might seem strange that, with all of the technical jargon associated with the various components (vectors, matrices, etc.) of the multiple alternatives model™, the term "right hand side" or RHS is a member of the technical dictionary. It may be the only term which inherently makes sense from genesis.

STRUCTURE OF THE CONSTRAINT MATRIX

The constraint matrix contains the full system of linear inequalities and equalities which will ultimately constrain the evaluation, comparison and final selection of some number of subsets of feasible solutions from among the range of alternatives available, and thus the rationale for the matrix's name. In structure, the constraint matrix is simply the condition and bounds vectors appended to the right-hand-side of the coefficient matrix.

Each row (i.e. vector) of the constraint matrix therefore represents a constraint to the modeled decisionmaking process, decisioning here defined as the selection of feasible alternatives from the set of all available alternatives. The secondary process of choosing the one, best alternative (usually a single subset of alternatives) will be discussed later when we explore the relationship of the objective function to the constraint matrix during model execution. For now, to better understand the structure of the constraint matrix, we offer the following figure.

		Multiple Decision Alternatives (X_j)							Cond Vect	Bound Value
Criteria (A_i)		X_1	X_2	X_3	X_4	X_5	...	X_j		
<-	Maximizer - Criterion A_1	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	...	a_{1j}	(lower bound) $\geq$	b_1
	Minimizer - Criterion A_2	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	...	a_{2j}	(upper bound) $\leq$	b_2
	Identity - Criterion A_3	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	...	a_{3j}	(identify bound) $=$	b_3
	$\vdots$	$\vdots$						$\vdots$	$\vdots$	$\vdots$

---> Maximizer Constraint:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 + \dots + a_{1j}x_j \geq b_1$$

illustrates a constraint whose summation must equal or exceed the value of the lower bound $b(1)$ found within the RHS; Maximizing constraints normally signify criteria of positive effect (i.e. benefit) which attempt to accrue maximum impact to the system based upon the mix of alternatives selected.

-----> Minimizer Constraint:

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 + \dots + a_{2j}x_j \leq b_2$$

illustrates a constraint whose summation may equal but never exceed the value of the upper bound $b(2)$ within the RHS; Minimizing constraints normally signify criteria of negative effect (i.e. cost) which attempt to limit the amount of negative impact to the system based upon the mix of alternatives selected.

-----> Identity Constraint:

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 + a_{35}x_5 + \dots + a_{3j}x_j = b_3$$

illustrates a constraint whose summation must equal (i.e. be identical to) the parametric value $b(3)$ within the RHS; Identity constraints normally signify primary control criteria which delimit the relationship between alternatives and criteria.

The constraint matrix as a system of linear inequalities and equations guides the decisioning process, informing the model that any subset of alter-

natives (dependent variables defined by the columns of the matrix) can be considered feasible (i.e. potentially a solution) if and only if the sum of the alternatives' respective values in each of the constraint matrix's row vectors (i.e. criteria) satisfy the condition and bounds set in the RHS. Therefore, if the constraint matrix contains 20 rows (20 criteria constraining the decisional process), then any subset of alternatives, to be considered as a feasible solution alternative, must produce individual row sums which simultaneously satisfy each of the 20 conditional bounds set in the RHS. The sheer mystique of such a simultaneous process, of course, pertains to the fact that different combinations of the alternatives will produce different sums, and therefore a variable potential for becoming a member of the set of all subsets of feasible solution alternatives.

Determination of the Decisional Space. Like most search, sort and select operations, the multiple alternatives model[™] must first determine, from all possible alternative actions (in our case, program funding level options), what combinations of alternatives simultaneously satisfy the constraints established by the constraint matrix. Since each of the constraint vectors of the constraint matrix compete for influence on the choice of what number of alternatives and which ones (when summed) satisfy the RHS, the presence of any particular vector is significant in determining whether a subset of the available alternatives is feasible or infeasible as a potential solution.

The concept of feasibility is sometimes confusing to the novice, since some subsets of alternatives will be termed feasible, others infeasible, and therefore some finite number of subsets of feasible alternatives as alternative feasible solutions, from which a single subset will later be selected as the optimal solution. All possible combinations of alternatives available to the model define what is called the decisional space. Some region inclosed within the decisional space must be defined as feasible, that is, containing those subsets of alternatives which satisfy all constraints. From this region of feasibility a single subset must be identified as optimal. From the mathematician's viewpoint, this is trivial. For the model builder, several hours of effort may be necessary unless care is taken initially.

Search for the Feasibility Region. In the hierarchical school closure model, not only must the various criterion characteristics (plant, instructional, etc.) of all campuses introduced into the model be 'balanced' in the final solution, and therefore in one-to-one correspondence with the overall program needs and policy mandates of the school district, but moreover all other 'constraints' to the decisioning process must also be 'balanced'. Constraints on effectiveness, efficiency, satisfaction, perception, etc. must all be balanced according to the dictates of the RHS. It is the RHS which partitions the region of feasibility from the decisional space, and identifies each of the subsets of feasible alternatives which are members of that feasibility region. Thus, the RHS provides the clue to determining initial feasibility in the school closure model.

It may be somewhat obvious to the reader, that if the conditional bounds of the RHS are too restrictive and therefore no subset of alternatives can be identified as satisfying all of the constraints in the model, no corresponding region of feasibility exists. Therefore, no solution exists, optimal or otherwise. This presents an interesting (actually nerve-racking) problem. If initial feasibility is not established on the first execution of the model, all the model can do is to simply report "feasibility not established". Unfortunately, the model can not suggest that if the designer will simply raise 'this' particular upper bound, lower 'that' particular lower bound, and change 'some' specific identity bound, an initial region of feasibility might be established. The model is lost, and so are you if you do not know what to do next.

A simple rule of thumb will preclude this difficulty getting out of hand. It may not prevent the problem from occurring initially, but you should be able to establish feasibility on at least the second (or at the most, third) try. The rule:

Apply extreme relaxation initially in the RHS by constructing very low 'lower bounds' for maximizer constraints, very high 'upper bounds' for minimizer constraints, and being selective in the use of identity constraints. This technique should establish a baseline feasibility

region from which you can gradually and individually restrict each of the constraints (by raising lower bounds, etc.) to model adequately and appropriately the situation intended for the model. When infeasibility is once again encountered, it is obviously the result of the last restriction imposed on the model. Relax this restriction to its previous level, and continue restrictions with another constraint as appropriate.

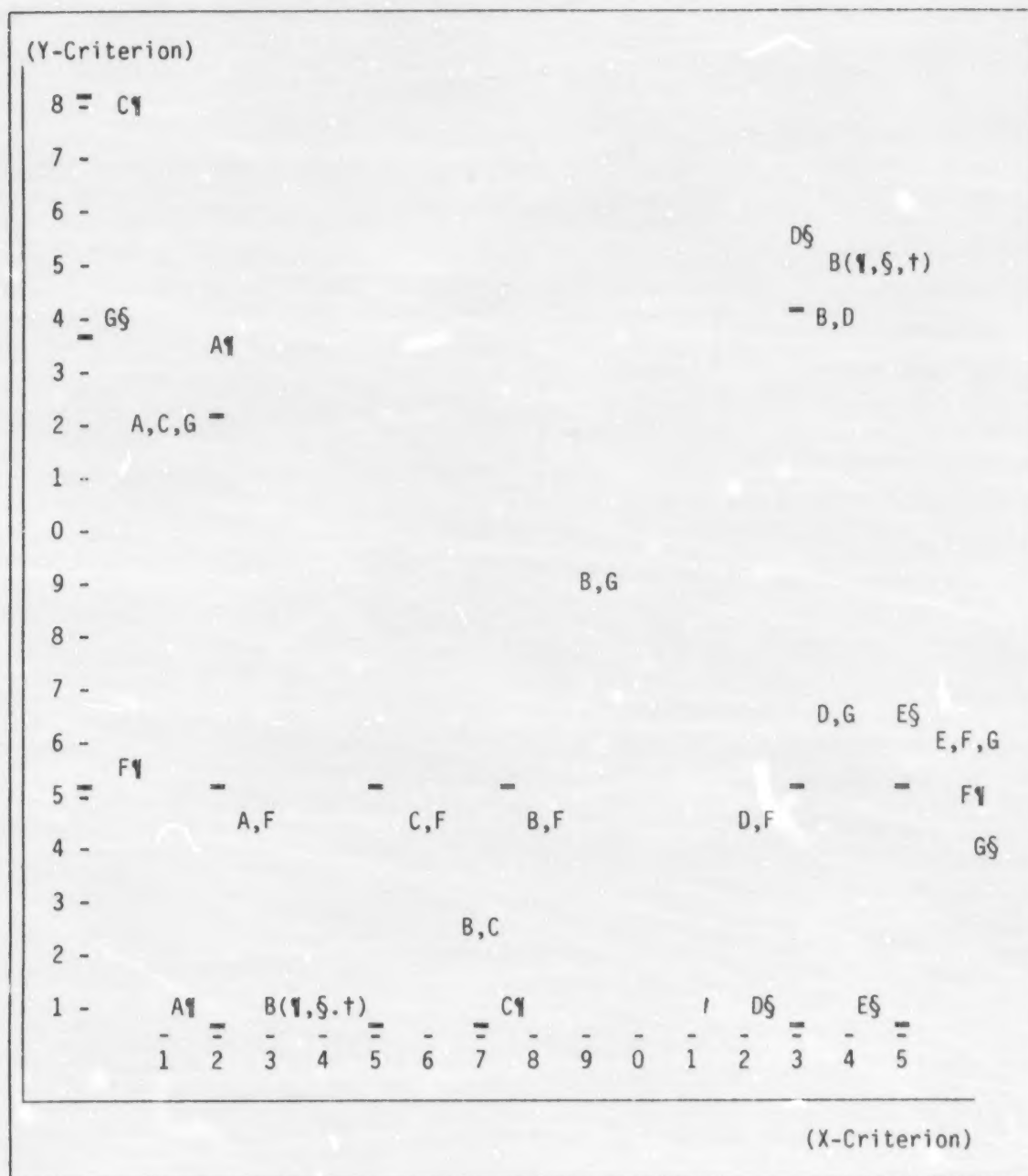
To better understand this procedure, and how it works, let us explore the geometric relationships the model draws upon to do it work.

Identification of Subsets of Feasible Alternatives. Since linear inequalities and equations can be plotted as x-y coordinates (two-dimensional space) when two alternatives are being compared or as x-y-z coordinates (three-dimensional space) when three alternatives are evaluated, it necessarily follows that graphing more than three alternatives is feasible though impractical. To solve this problem for the reader, a technique called composite mapping will be employed. Mapping simply means to construct a visual representation of a mathematical formulation (e.g. the graph of a linear inequality on coordinate graph paper). Composite mapping means to switch the scaling of the graph from the normal dependent/independent variable scales to axes which represent a composite of the functions graphically depicted utilizing these scales.

Normally, a multiple alternatives problem of two alternatives can be depicted on simple graph paper by having the x-axis represent one of the alternatives as an independent variable, and the y-axis represent the other alternative as a dependent variable -- if we allow the value of the alternatives to be scaled as continuous (distribution) variables. This same technique works for three dimensions as well when three alternatives are being evaluated. In our model, the problem is that we wish the dependent variables to take on values of '1' or '0', only, representing their membership or nonmembership in the solution set, respectively. We will circumvent this problem by defining our axes to represent continuous distributions used to scale two criterion constraints in the model. Vector graphs of inequalities and equalities will represent each of

the alternatives being compared, the singular influence on each alternative by the two criteria, and each alternative's composite proximity to the region of feasibility.

Consider the following graph, in which seven (7) alternatives are graphically represented by means of the corresponding influences of two (2) criterion variables. Vectors A_{ij} , C_{ij} and F_{ij} are defined as representing maximizing constraints, while vectors D_{ij} , E_{ij} and G_{ij} are defined as representing minimizing constraints. Vector $B(i, j, t)$ may take on the constraint qualities of maximizer, minimizer or identity depending upon the dictates of the model developer. All vector intersections identified by the letters A,...,G demonstrate areas where the model has calculated the relative worth of a single alternative, or group of alternatives, for its (their) potential as members of the solution.



(† maximizer constraint; § minimizer constraint; † identity constraint)

where:

X-Criterion (Scale Interpretation)

Interval-1			Interval-2			Interval-3			Interval-4			Interval-5		
01	02	03	04	05	06	07	08	09	10	11	12	13	14	15
LO	MD	HI	LO	MD	HI	LO	MD	HI	LO	MD	HI	LO	MD	HI

Example: '05' rates the alternative as INTERVAL-2/MEDIUM on Criterion X.

and,

Y-Criterion (Scale Interpretation)

Interval-1			Interval-2			Interval-3			Interval-4			Interval-5			Interval-6		
01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18
LO	MD	HI	LO	MD	HI	LO	MD	HI	LO	MD	HI	LO	MD	HI	LO	MD	HI

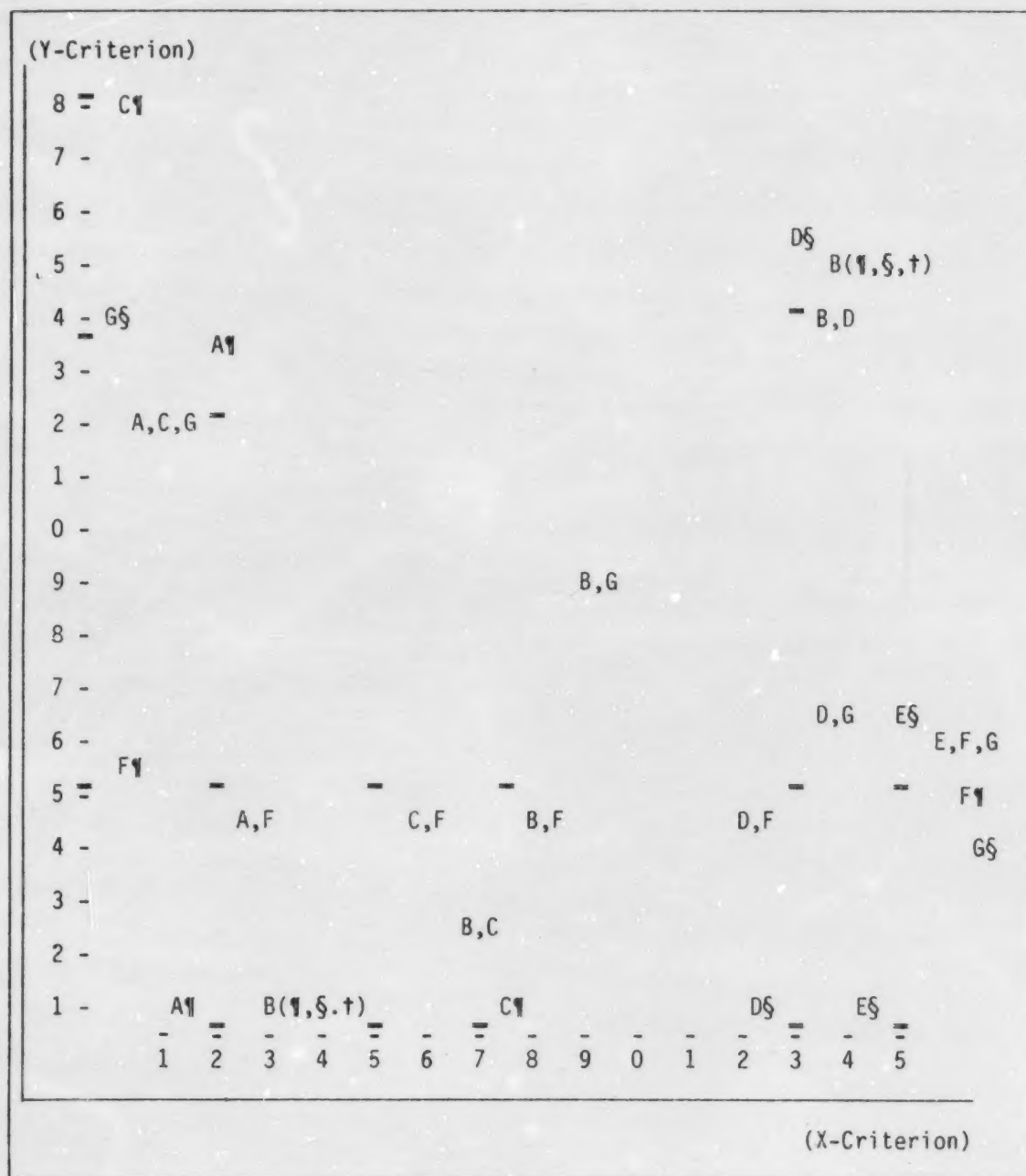
Example: '15' rates the alternative as INTERVAL-5/HIGH on Criterion Y.

Ignoring vector $B(\mathbf{1}, \mathbf{5}, \mathbf{1})$ for now, let us assume that the region defined by the vertices $[(A,C,G), (C,F), (B,F), (D,F), (D,G), (B,G)]$ satisfies all constraints represents by the three maximizer and three minimizer constraints. By extension, we can assume that each vertex, any point on any vector bounding this region, and any point within the region bounded by the four surrounding vectors also satisfies all six constraints, simultaneously. This region is the region of feasibility discussed earlier. (See outlined portion of graph on preceding page.) Actually, we need only concern ourselves with the vertices of the bounded region, since mathematical proof demonstrates that only these subsets of identified alternatives will prove to be optimal when the objective function is applied. (We will not concern ourselves with proving this statement here.)

We will now explore the effect upon the existing region of feasibility which the vector $B(\mathbf{1}, \mathbf{5}, \mathbf{1})$ will have depending upon whether it is chosen as a maximizer, minimizer or identity constraint. For discussion purposes, we will assume that if B is maximized, then the area to the right of the vector (including all points contained on the vector itself) are feasible. On the

other hand, if B is minimized, then the area to the left of the vector (again including all points on the vector) are feasible. However, if B is identified as an identity constraint, then only the points on the vector line itself can be considered to be feasible, all points to both the right and left of the vector not satisfying the constraint's of equality.

Therefore, if B_{η} exists, then the feasibility region is decreased to include only that sub-region defined as $[(B,G),(B,F),(D,F),(D,G)]$. On the other hand, if B_{ξ} exists, then the subsequent feasibility region is decreased to include only the space $[(A,C,G),(C,F),(B,F),(B,G)]$. What happens if B_{τ} exists? With B_{τ} , only that part of the original $[(A,C,G),(C,F),(B,F),(D,F),(D,G),(B,G)]$ which is also B_{τ} is rendered feasible. Thus, only the points on line $[(B,F),(B,G)]$ can be considered feasible. Therefore, depending upon the condition imposed by $B(\eta,\xi,\tau)$, the region of feasibility may defined only four, four and two feasible combinations of alternatives, respectively.



Choosing which of these combinations of feasible alternative subsets is, in fact, optimal is the responsibility of the objective function.

CONSIDERATIONS FOR SCANT SUBMATRICES

The scant matrix is different from that of the zero matrix. A zero matrix is a rectangular set of coefficients, all of which are the value zero. In most cases, such an occurrence simply means that this zero matrix is a significant portion of a larger matrix with at least one nonzero value. In mathematics, on the other hand, the occurrence of a zero matrix is often the subject of a proof or theorem. The scant matrix is defined as a matrix (or submatrix) where a significant (left to the reader's interpretation) number of coefficients are of the value(s) zero or empty (viz. without value). This statement is not trivial.

Take the case of a submatrix which has cost-related values as coefficient entries. If a program does not required a certain allocation of account or fund specific monies, then the coefficient value relating that account (or fund) to the program is empty. In the case of the hierarchical school closure model, the criterion of average walking distance from attendance areas to a particular school is nongermane if the school option is 'closure', and therefore the coefficient of average-miles-walked would not be posted for any of the three closure options available. In mathematics, of course, we normally substitute a value of zero when no other value exists. As a matter of fact, the computer will automatically entry a value of zero in all cases where no other value was defined. This poses a problem albeit resolvable for the modeler when initially building the coefficient matrix, and later when computing reliable RHS-values for the bounded limits vector. This section will discuss these problems in detail, and offer a reasonable solution to the situation.

Definition of the Scant Submatrix. A matrix/submatrix is scant if a large share of the resident coefficients in the matrix can not be assigned a valid value because of the particular set of criteria being measured for the multiple alternatives existing. The distinction here is between a value of zero (as an interval or ratio scaled measure) which means a lack of a measurable amount, the

value of 'empty' which means the inability to obtain a measure for the various dependent variables across the criteria listed. Arithmetically, if a vector of values contain zeros (of the first case) then the zeros can be averaged with the remaining nonzero vector coefficients to produce a mean. On the other hand, a series of empty entries imply values of nonquantification, and therefore can not be used in the averaging process. Unfortunately, the computer will substitute zeros for any value which is not directly and numerically entered.

The Ability to Discriminate Coefficients in the Scant Matrix. The model builder must have the capacity to discriminate (tell the difference) between zero coefficients and empty cells which have been filled by the computer with zeros automatically. Luckily, this ability is easily realized, with a little assistance from number theory which caused the problem in the first place. First of all, all zero coefficients -- values which necessarily portray measures of zero weight -- should and must be entered as zeros. That is the easy part. Entries for empty coefficients take on one (1) of two different values depending upon whether the vector of the scant submatrix will be subsequently defined as a maximizer or a minimizer. (We will deal with identity constraints separately.)

Under constraint maximization, the sum of vector coefficients attempts to grow as large as possible while in competition with the other constraints of the constraint matrix. A lower bound has already been defined in the BLV of the RHS. To assure that the model recognizes particular coefficients as empty rather than zero the value for each of these coefficients should be set at "negative infinity", or typically the value -999999999E99 for a significant digits system of 9-digits. Similarly under constraint minimization, the sum of vector coefficients attempts to grow as small as possible. With an upper bound established in the BLV for this constraint, an empty coefficient should be represented as "positive infinity", or the value +999999999E99 in the 9-digits (significance) system. Why does such a convention work? We will prove the case for maximization, leaving the corollary case under minimization for the reader to attempt.

Consider the vector coefficients of the following constraint:

$$1000 \quad 3000 \quad (\quad) \quad 2000 \quad (\quad) \quad 0000 \quad 0000 \quad (\quad) \quad 2000 \quad 1000 \quad \geq \quad 5000$$

where the designation '()' represents an 'empty' (not zero) coefficient. To differentiate between the zero (0000) entries and these empty entries, the value of -9E9 (approximating negative infinity) will be substituted for each empty '()' coefficient. This is represented by the new vector:

$$1000 \quad 3000 \quad -9E9 \quad 2000 \quad -9E9 \quad 0000 \quad 0000 \quad -9E9 \quad 2000 \quad 1000 \quad \geq \quad 5000.$$

In operation, the model will systematically ignore any of the -9E9 entries for their potential contribution to the sum and comparison with BLV = 5000, since the choice of even one of the -9E9 values would prevent the sum of the remaining zero and nonzero coefficients from ever reaching the '5000 cut-off'. Thus, the model will survey only non-(-9E9) values as it searches for feasible solution subsets.

In the case of the identity constraint, the choice between +9E9 and -9E9 values for empty coefficients is usually inconsequential. If only positive non-zero coefficients are evident in the vector, use +9E9 always. This is always the guideline for vectors which have only negative coefficient values. When a mixture of positive and negative coefficients are resident in the vector, use a value which is -9E9. We leave the proof to the reader.

The Ability to Compute BLV in the RHS for the Scant Matrix. A little common sense goes a long way when calculating BLV for either maximizer or minimizer constraints. Recall that the computation of lower and upper bounds is a function (normally) of the difference between the mean and standard deviation of the vector's coefficients multiplied by some integer factor for lower bounds, and the sum of the mean and standard deviation of the vector's coefficients multiplied by some integer factor for upper bounds. To compute BLV in the scant matrix, the rule is simple: include only nonzero and zero coefficients in the sum and averaging -- ignoring the positive (or negative) infinities representing empty coefficient entries. In the case of the identity, the BLV is typically

supplied by the decisionmaker (e.g. total school district budget available for allocation and distribution), and is not computed by the modeler.

The Special Case of Vectors in the Control Submatrix. Our concern for zero and empty entries in the coefficient matrix does not extend to the submatrix in which our 'alternative-comparison control' vectors are stored. Wherever there are no 1's designating which alternatives are to be considered, the value of 0 will reasonably suffice. One could, of course, enter a +9E9, but this would be wasting the valuable central memory of the computer -- an especially noteworthy point is the model is using a microcomputer.

STRUCTURE OF THE OBJECTIVE FUNCTION

The last component of the multiple alternatives modeling™ framework, the objective function, provides the model with the ability to choose the one, best solution subset from all subsets of feasible solution alternatives identified by the constraint matrix. The objective function is itself a criterion vector, but without a formal conditional bound reflected in the RHS. The values in the objective function are summed in exactly the same way as are the vector elements in the coefficient matrix during the model's comparison and contrast operations. With no RHS-value attached, the objective function simply attempts to either maximize or minimize its summation. That subset of alternatives which is determined to be (1) feasible and (2) present a sum of objective function entries which is highest or lowest, depending upon whether the objective function is to be maximized or minimized respectively, is considered the optimal solution (subset).

Viewed structurally as part of the MAM™ framework, the objective function completes the model's full design, and can be illustrated as follows.

CONSTRAINT MATRIX

school being evaluated ->	School Alternative j, j = 1,...,n				
Level of closure (if any) ->	CLOS1	CLOS2	CLOS3	OPEN1	OPEN2

Positive Effects Criteria => criteria which measure desirable influences across alternatives	Positive Effects Sub-Matrix (maximizer constraints w/ lower bounds)				
Negative Effects Criteria => criteria which measure undesirable influences across alternatives	Negative Effects Sub-Matrix (minimizer constraints w/ upper bounds)				
Modeling Control Criteria => criteria which provide control to hierarchical characteristics of model	Control Sub-Matrix (identity constraints w/ identity bounds)				

Optimality Criterion =====> single vector whose sum allows model to select subset of alternatives which are both feasible and optimal (best subset)	c1x1 + c2x2 + ... + cjxj, j = 1,...,n				

OBJECTIVE FUNCTION

The geometric role of the objective function (sometimes called the optimality vector) is to select which of the vertices of the feasibility region produce the maximum (or minimum) sum, depending upon whether the criterion basis for the objective function is positive or negative, respectively. In two-dimensional space (two alternatives being compared), the objective function is a line passed through the feasibility region searching for a vertex. In three-dimensional space (three alternatives evaluated), the objective function is a plane similarly passed through a region which is solid (three-dimensional). The point at which the objective function (line or plane in these examples) commences to depart the region of feasibility declares either the function's minimum or maximum value, and thereby the optimal solution based upon the identity of the vertex.

A number of additional issues, however, must first be addressed before leaving this topic, and proceeding to a more specific illustration of a hierarchical school closure modeling formulation in MAM™.

Components of the Singular Optimization Approach. The successful execution of the model needs only a single objective function to guide the final choice from among the varying feasible subsets of decisional alternatives. Therefore, the model developer must recognize inherent problems in the use of an objective function whose cell entries (coefficients) vary only slightly. As the sum of the difference between the computed mean of the coefficients of the objective function and each of these same coefficients approaches zero, the model must rely more heavily upon the maximizer, minimizer and/or identity constraints in the constraint matrix. Several problems do arise if the constraints are unable to significantly discriminate between the identified subsets of feasible alternatives, when the objective function has computed equal sums for several of these alternatives.

The use of a single objective function produces a single optimal solution if a feasible solution exists, all other things remaining equal. In order to produce a second (or third, etc.) optimal solution for comparison to the first, the only recourse is to relax and/or restrict the RHS of one or more constraints. Oftentimes however, this simply produces the same optimal solution as before. To construct a separate optimal solution under singular optimization, the relaxation and/or restriction of constraints results in a model which is neither reasonable nor valid to the problem at hand. The alternative to this dilemma is to employ a cyclic optimization technique.

Components of the Cyclic Optimization Approach. Cyclic optimization employs two or more (typically more) objective functions, executing each in turn, and producing separate results which can be compared and contrasted by various statistical techniques. Research by the author has demonstrated that even the use of some of the criterion vectors in the coefficient matrix (i.e. re-use) can produce variable optimal solutions which can then be post-hoc analyzed for their reasonableness as an actual final solution for implementation.

The Threat of Multicollinearity. Model builders are wise to run simple correlations among the vector coefficients which will become constraints, and between the criterion coefficients and the coefficients of the objective function. If the coefficients of two (or more) vectors of the coefficients matrix produce significantly high correlations, it would be reasonable to delimit one from further use, or to form a composite of the vectors involved. Multicollinearity within the coefficient matrix is inefficient since highly related values are being modeled together. At worst, such vectors have the potential to produce degenerate solutions.

A significantly high correlation between the objective function and one or more of the vectors of the coefficient matrix introduces decisional bias into the model which may not be intended, or if intended, must be acknowledged.

STEPWISE PROCEDURES FOR TARGETING SUCCESSIVE SCHOOL CLOSURES

The hierarchical school closure model introduces another 'special consideration' in modeling, in addition to the scant submatrix, which might not be germane to other models. This is the process of stepwise decision selection.

The closure model simultaneously determines (1) what number of schools will be closed, (2) the disposition of those closed schools, and (3) what will be the operating level of those schools remaining open. However, the model on its first pass may (quite reasonably) close two adjacent schools -- one of which might have fared better during the evaluation if it could have laid claim to the student enrollment of the other school. When closing a school, it is reasonable to expect the decisionmaker to transfer students of the closed school to a neighboring school (or schools). Such transfer, and the additional monies now available to the district, might significantly and positively impact an adjacent school, and preclude its being closed in the future. Therefore, the model must be informed where the students of a closed school will be transferred, and must have its coefficient matrix and RHS updated to reflect the new state of the district. Such requirements could be modeled, but the process is cumbersome. An alternative is available.

From the first results of the model, choose a single school for closure, transfer its student, update the constraint matrix, and run the model again, this time with one less column in the coefficient matrix. From the next set of results, choose a second school, and continue the process until the model tells you 'no further closures are necessary' (i.e. infeasibility is declared). Typically, and especially for a larger school district with several schools, the choice of a single school for closure is a relatively clear process.

THE HIERARCHICAL SCHOOL CLOSURE MODEL

Our rather laborious and circuitous route will hopefully better enable the reader to understand and accept the complexity and comprehensiveness of the hierarchical school closure model. Precisely stated, this model seeks to:

TARGET THE NECESSARY NUMBER OF SCHOOL CAMPUSES IN THE SCHOOL DISTRICT FOR FUTURE OPERATIONAL DISCONTINUANCE OR CONTINUANCE AND THE SPECIFIC LEVEL OF THAT DIS/CONTINUANCE, RESPECTIVELY (choosing from three pre-evaluated options for closure (i.e. including sale, leasing or mothballing) and two options for operational continuance (functional partitioning with sub-site leasing and full program continuance (no change)

subject to:

1. maximizing the positive influence from all school-based programs, and their support administrative and physical plant structures, on total school district functioning, based upon final school-by-school closure levels selected;
2. minimizing the negative influence from all school-based programs, and their support administrative and physical plant structures, on total school district functioning, based upon final school-by-school closure levels selected;
3. assuring that all schools and their programs within the school district are nondiscriminantly evaluated, compared and contrasted to ascertain their contributing value or worth to the goals of the school district as a whole; and,

4. conclusively targeting levels of operational discontinuance or continuance based upon valid measures of effectiveness, expenditure, economy, efficiency, and satisfaction pertinent to the character of each school, and its instructional mission.

To accomplish this intent, a reformulation of the multiple alternatives modeling™ framework (Wholeben, 1980) for hierarchical decisionmaking has been explored. Within the MAM™ context, the operational alternatives for each school are delineated into five (5) closure/non-closure options depending upon a pre-modeling evaluation of each school, and the resulting school options scenarios or narratives which result. Criteria, in the form of numerical vectors, are introduced within the model to (1) depict the pluses and minuses associated with each 'school closure' alternative, (2) compete for influence in the comparison and contrast of each 'closure' alternative with other likely options, and then (3) simulate the decisional environment of the school district, and its various service programs and/or instructional departments. This modeling approach to evaluating and selecting variant school closure alternatives based upon merit and estimated impact is MAM™ formulated as follows:

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The reader should notice that the above depiction incorporates the use of cyclic optimization techniques for model execution. It is the author's view that cyclic optimization is superior to singular optimization (the use of a single objective function during alternatives evaluation) because of the additional information available for comparing and contrasting varying optimal solutions. Statistical techniques are available for performing such post-hoc analyses.

A diagrammatic insert (following this page) symbolically demonstrates the use of a criterion-based focus for modeling mandated policy guidelines (limits) across likely alternative school operational discontinuance or continuance scenarios. Although only ten criterion foci are modeled across the various school sites, each with fixed-level hierarchical closure options, it is more likely to expect a constructed model to evaluate a hundred or more criterion influences across the full complement of school sites in the district.

Luckily, computer-based analysis programs are available to preclude the necessity for most hand calculations. The author is currently constructing such a computer program which will be interactive and executable on small, readily available microcomputers, such as the Apple IIe.

As previously discussed at length, the criterion based influence within the model allows superior flexibility to the modeler for introducing the effect of certain influences on the final selection of a best-match (optimal) solution. Criteria reflecting measures of program effectiveness, expenditure, economy, efficiency and satisfaction are able to be input to the model for comparing and contrasting the individual impact of any one closure option, and that closure's contributing impact to the school district. These criterion influences may be based upon physical, transformation, definitional and/or synthetic datum bases, and entered into the model with any one or more of the typical nominal, ordinal, interval or ratio scale formats.

These criteria compete with each other for their influence in selecting the final solution -- a targeted closure or operational continuance level for each school site modeled.

School-A	School-B	School-C	School-D	School-E	School-F	School-G	School-H	School-I	
									total district budget
									current district inventory
									projected district needs
									maximum negative impact
									minimum positive impact
									district integrated programs
									maximum district consensus
									minimum district policy
									maximize district goals
									each district program

To illustrate this criterion scaling, observe the following figure.

	School Plant #1						School Plant #j							
	C/1	C/2	C/3	O/1	O/2	...	C/1	C/2	C/3	O/1	O/2			
Plant ADM Capacity	c_1					c_1	...	c_j					c_j	$\geq$ Futur Distr ADM
	(expressed in students-count, from architectural blueprints)													
Plant ADA Vacancies	v_1					v_1	...	v_j					v_j	$\geq$ compd lower bound
	(expressed in difference: plant capacity - plant ADM)													
School Prog Budgets	b_{14}					b_{15}	...	b_{j4}					b_{j5}	$\leq$ Distr Budg
	(expressed in thousands of dollars, may be delineated by account)													
Anticipated Revenues	r_{11}	r_{12}	r_{14}		...	r_{j1}	r_{j2}	r_{j4}				$\geq$ compd lower bound		
	(expressed in thousands of dollars, may be delineated by source)													
Estimated Walk Dist	d_{11}	d_{12}	d_{13}	d_{14}	d_{15}	...	d_{j1}	d_{j2}	d_{j3}	d_{j4}	d_{j5}	$\leq$ compd upper bound		
	(expressed in average miles of site from attendance zones)													
Policy(s) Compliance	p_{11}	p_{12}	p_{13}	p_{14}	p_{15}	...	p_{j1}	p_{j2}	p_{j3}	p_{j4}	p_{j5}	$\geq$ compd lower bound		
	(expressed in frequency counts gathered through the scale below)													
	Fails to Meet Most Requirements [1]		Fails to Meet Some Requirements [2]		Marginally Acceptable (Neutral) [3]		Meets Most Requirements as Defined [4]		Exceeds Most Requirements as Defined [5]					
Opinion from Survey(s)	o_{11}	o_{12}	o_{13}	o_{14}	o_{15}	...	o_{j1}	o_{j2}	o_{j3}	o_{j4}	o_{j5}	$\geq$ compd lower bound		
	(expressed in frequency counts gathered through the scale below)													
	High Negative [1]		Negative [2]		More NEG Than POS [3]		More POS Than NEG [4]		Positive [5]		High Positive [6]			
Administrator Preference	a_1	a_1	a_1	a_1	a_1	...	a_j	a_j	a_j	a_j	a_j	$\geq$ compd lower bound		
	(expressed in frequencies, respondents indicating preferred sites)													
Control #1	1	1	1	1	1							= 1		
Control #2							1	1	1	1	1	= 1		

The reader's attention is called to a number of interesting albeit important contrasts between vectors in the above model. First, note that the first three criterion constructs (capacity, vacancy and budget) define coefficient matrix values for only the 'open' alternatives, while the fourth criterion constraint defines values for a subset of both 'close' and 'open' alternatives. In the first case, the importance of plant capacities, site vacancies, and required operating budgets is only germane if the plant/site is operating. Thus, these values need only be represented for the 'open' alternatives. Similarly, since revenues can only be expected if and only if the plant is available for some form of leasing or sale, only the first, second and fourth alternatives are germane to this standard.

In the case of the criterion 'estimated walking distance', a seemingly contradictory constraint is offered wherein walking distance is defined for the closure options as well as the open alternatives. Such a strategy would be necessary is the modeler wish to directly target schools which had relatively large walking distances associated with it, rather than simply target schools for 'nonclosure' based upon relatively small walking distance measures.

While many other illustrations and/or examples are possible, the widely varying environment for targeting potential school closures nearly precludes the design of a 'conventional model' which would readily fit every school district's needs. The flexibility with which the model can be designed to fit very individualistic needs is both benefit and pitfall. While flexibility allows a model to conform strictly to needs peculiar to any individual modeling situation, this same flexibility forces the developer to re-think the model's formulation for each new school closure situation, especially as characteristics of supply and demand change to fit the fluid needs of a highly variant society.

SUMMARY AND RECOMMENDATIONS

Historically, the MAM™ approach has been applied to elementary school closures and budgeting reductions within public school districts. Other models which have been developed include: (1) matching instructional activities with

market-available microcomputer hardware and software; (2) consolidation and merger of rural school districts; and (3) the reconfiguration of primary, elementary and secondary grade levels, by building, for medium-sized school districts.

The model's initial formulation in 1978 identified a total of five (5) elementary schools from a field of 32 schools for closure. Further research using the graduated budget reduction model during 1981-83 demonstrated that the model effectively and efficiently targeted the appropriate programs and their respective level of reduced funding, based upon the criterion references supplied to guide the decisionmaking process. Post-hoc statistical analyses which matched the data used to the results generated significantly validated the model's technique and simulation processes.

Certainly, no computer simulation technique or mathematical model should be placed in a position to make the decision for the district administrator in the case of targeting district schools for closure. However, this is not to say that such a data analysis and alternatives evaluation technique should not be used to compare the multiple of options available to the decisionmaker, and to provide more composite information for the final decision process. If school closure is a rational albeit controversial option, the decisionmaker might at least desire to know "If so, which ones and how many?".

The multiple alternatives modeling™ approach to targeting schools for potential closure provides this necessary information.

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